PAPER

Relations between Common Lyapunov Functions of Quadratic and Infinity-Norm Forms for a Set of Discrete-Time LTI Systems

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SUMMARY This paper studies the problem of the relations between existence conditions of common quadratic and those of common infinity-norm Lyapunov functions for sets of discrete-time linear time-invariant (LTI) systems. Based on the equivalence between the robust stability of a class of time-varying systems and the existence of a common infinity-norm Lyapunov function for the corresponding set of LTI systems, the relations are determined. It turns out that although the relation is an equivalent one for single stable systems, the existence condition of common infinity-norm type is strictly implied by that of common quadratic type for the set of systems. Several existence conditions of a common infinity-norm Lyapunov functions are also presented for the purpose of easy checking.

key words: *common quadratic Lyapunov function, common infinity-norm Lyapunov function, robust stability, set relations, existence conditions, discrete-time systems*

1. Introduction

The common Lyapunov function problem is a problem that investigates the existence of a common Lyapunov function for a set of linear time-invariant (LTI) systems. The problem frequently arises in stability analysis and control design of various types of control systems such as uncertain systems [1], switched systems [6], fuzzy systems [20], etc. This makes the problem one of basic issues relating to the stability analysis and control design of these systems. For a comprehensive survey, see e.g., [12], [15]. Given numerical data, the existence is checkable using existent software packages, but analytical properties for that are yet to be fully explored.

Presently, there are two types of common Lyapunov functions which have been studied to considerable extent: common quadratic Lyapunov function (CQLF) and common infinity-norm Lyapunov function (CILF). The quadratic Lyapunov functions have been studied deeply and appear to be a powerful tool in investigating the system stability problem. Correspondingly, the CQLF problem has been researched in [2], [10], [11], [13], [18], [19], etc. The reason of the popular presence of CQLF is clear due to the existence of powerful tools for their use such as Lyapunov matrix inequalities and LMI (Linear Matrix Inequalities) packages. Recently, the Lyapunov functions of the form of the vector infinity-norm have also received a lot of attentions. These infinity-norm Lyapunov functions have been used for robust stability analysis [4], [7], [8], [16], [17] and the associated CILF problem has been studied in [12], [15] and so on. With these situations in sight, the relations between CQLF and CILF for sets of continuous-time LTI systems are clarified in [15]. However, the question whether the corresponding results for the discrete-time case hold or not still remains open and it is of interest to examine the relations. In this paper, we investigate the discrete-time counterpart of the problem. It turns out the relations hold in parallel manner between both of the cases. Several sufficient existence conditions of a CILF are also presented.

The content of this paper is organized as follows. Section 2 states the preliminaries of the common Lyapunov function problems and some supporting facts about the robust stability conditions of a class of discrete-time linear time-varying systems. Main results are given in Sect. 3, where we clarify the relations between CQLF and CILF and provide several sufficient existence conditions of a CILF. The conclusion is given in Sect. 4.

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2. Preliminaries

Following symbol convention is utilized throughout the paper. Let $\mathcal{R}^n$ denote real n dimensional space. If $\mathbf{x} \in \mathcal{R}^n$, then $\mathbf{x}^T = [x_1, ..., x_n]$ denotes the transpose of $\mathbf{x}$. Let $\mathcal{R}^{m\times n}$ denote the set of $m \times n$ real matrices. With $\lambda(\mathbf{A})$, $\mathbf{A} \in \mathcal{R}^{n\times n}$, the spectrum of $\mathbf{A}$ is expressed. If $\mathbf{A} = [a_{ij}] \in \mathcal{R}^{m\times n}$, then $\mathbf{A}^T$ denotes the transpose of $\mathbf{A}$. We let $||\mathbf{x}||$ stand for any one of the equivalent vector norms on $\mathcal{R}^n$. In particular, the l_p norms $||\mathbf{x}||_p$, $1 \le p \le \infty$, are defined by

$$||\mathbf{x}||_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}, \; 1 \le p < \infty,$$

and

$$||\mathbf{x}||_\infty = \max_{1\le i\le n} |x_i|.$$

The matrix norm $||\mathbf{A}||$, defined on $\mathcal{R}^{n\times n}$, induced by a vector norm $||\mathbf{x}||$ in $\mathcal{R}^n$, is defined as

$$||\mathbf{A}|| = \max_{||\mathbf{x}||=1} ||\mathbf{A}\mathbf{x}||.$$

In particular, the infinity norm of matrix $\mathbf{A}$ is defined by

$$||\mathbf{A}||_\infty = \max_{||\mathbf{x}||_\infty=1} ||\mathbf{A}\mathbf{x}||_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}|.$$

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2.1 Common Lyapunov Function Problems

We first state definitions of CQLF and CILF [12]. Consider a set of stable discrete-time LTI systems described by the following equations

$$\mathbf{x}(t+1)=\mathbf{A}_i\mathbf{x}(t),\ \mathbf{x}\in\mathcal{R}^n,\ \mathbf{A}_i\in\mathcal{R}^{n\times n},\ i=1,...,q. \tag{1}$$

Definition 1 The set of systems (1) is said to have a CQLF if there exists a symmetrical positive definite matrix $\mathbf{P}=\mathbf{P}^T>0$ such that the following Lyapunov inequalities

$$\mathbf{A}_i^T\mathbf{P}\mathbf{A}_i-\mathbf{P}<0,\ i=1,...,q \tag{2}$$

are satisfied and the CQLF is $\mathbf{V}(\mathbf{x})=\mathbf{x}^T\mathbf{P}\mathbf{x}$.

Definition 2 The function of the vector norm form $\mathbf{V}(\mathbf{x})=\|\mathbf{W}\mathbf{x}\|_\infty,\ \mathbf{W}\in\mathcal{R}^{m\times n},\ m\geq n,\ \mathrm{rank}(\mathbf{W})=n$ is said to be a CILF for the set of systems (1) if there exist matrices $\mathbf{Q}_i\in\mathbf{R}^{m\times m},\ i=1,...,q$ such that we have the matrix relations

$$\mathbf{W}\mathbf{A}_i=\mathbf{Q}_i\mathbf{W},\ i=1,...,q \tag{3}$$

$$\|\mathbf{Q}_i\|_\infty<1,\ i=1,...,q. \tag{4}$$

The set of matrices satisfying the conditions in Definition 1 (Definition 2) is denoted by $\mathcal{L}_Q$ ($\mathcal{L}_I$).

2.2 Some Supporting Facts

To show some supporting facts, we consider the following class of discrete-time linear time-varying systems

$$\mathbf{x}(t+1)=\mathbf{A}(t)\mathbf{x}(t),\ \mathbf{x}\in\mathcal{R}^n,\ \mathbf{A}(t):\mathcal{R}\to\mathcal{R}^{n\times n}, \tag{5}$$

with the time-varying matrix $\mathbf{A}(t)$ chosen arbitrarily from a polytope (convex combinations) of matrices

$$\mathbf{A}(t)\in\mathcal{A}:=\mathrm{co}\{\mathbf{A}_1,...,\mathbf{A}_q\} = \left\{\sum_{i=1}^{q}\alpha_i\mathbf{A}_i:\alpha_i\geq 0,\ \sum_{i=1}^{q}\alpha_i=1\right\}, \tag{6}$$

where $\mathbf{A}_1,...,\mathbf{A}_q$ are given fixed matrices.

We also consider the following multi-valued vector-function $\mathbf{F}(\mathbf{x})$ defined for all $\mathbf{x}\in\mathcal{R}^n$ by

$$\mathbf{F}(\mathbf{x})=\left\{\mathbf{y}:\mathbf{y}=\left(\sum_{i=1}^{q}\alpha_i\mathbf{A}_i\right)\mathbf{x},\alpha_i\geq 0,\sum_{i=1}^{q}\alpha_i=1\right\}. \tag{7}$$

Definition 3 The time-varying system (5) is said to be robustly stable with respect to the set $\mathcal{A}$ defined by (6) if its zero solution $\mathbf{x}(t)\equiv 0$ is globally asymptotically stable for any time-varying matrix $\mathbf{A}(t)\in\mathcal{A}$.

We state some known robust stability conditions for the discrete-time linear time-varying system (5) from [8] as follows:

Lemma 1 The time-varying system (5) is robustly stable with respect to the set $\mathcal{A}$ if and only if there exists a full column rank matrix $\mathbf{W}\in\mathcal{R}^{m\times n}$, $m\geq n$ with one of the conditions i) (Theorem 4.3 in [8]) and ii) (Corollary 4.6 in [8]) as follows:

i) There exist $m\times m$ matrices $\mathbf{Q}_i$, $i=1,...,q$ satisfying conditions (3) with (4).

ii) There exist a finite integer $p\geq 1$ and a constant θ ($0<\theta<1$), such that the Lyapunov function $V_{\mathbf{W},p}(\mathbf{x})$ defined by

$$V_{\mathbf{W},p}(\mathbf{x})=\sum_{i=1}^{m}(\mathbf{w}_i\mathbf{x})^{2p}=(\|\mathbf{W}\mathbf{x}\|_{2p})^{2p} \tag{8}$$

satisfies

$$\max_{\mathbf{y}\in\mathbf{F}(\mathbf{x})}V_{\mathbf{W},p}(\mathbf{y})\leq\theta V_{\mathbf{W},p}(\mathbf{x}), \tag{9}$$

where $\mathbf{w}_i$, $i=1,...,m$, is the i-th row of $\mathbf{W}$, and $\mathbf{F}(\mathbf{x})$ can be compactified without loss of generality.

Note that these two conditions are equivalent.

We will also invoke a known result involving the quadratic Schur stability of polytopes of matrices from [9] as follows:

Lemma 2 A necessary and sufficient condition for the existence of matrix $\mathbf{P}=\mathbf{P}^T>0$ such that $\mathbf{A}^T\mathbf{P}\mathbf{A}-\mathbf{P}<0$ $\forall\mathbf{A}\in\mathcal{A}$ is that there exists a common matrix $\mathbf{P}=\mathbf{P}^T>0$ and $\mathbf{A}_i^T\mathbf{P}\mathbf{A}_i-\mathbf{P}<0$, $i=1,...,q$.

3. Main Results

3.1 Relations between CQLF and CILF

In this section, we investigate the relations between CQLF and CILF for the set of discrete-time LTI systems (1).

Theorem 1 Given the set of systems (1), the existence of a CQLF strictly implies that of a CILF ($\mathcal{L}_Q\subset\mathcal{L}_I$).

Proof: By Definition 2, the equations (3) with (4) define the condition for the set of linear time-invariant systems (1) to share a CILF $V(\mathbf{x})=\|\mathbf{W}\mathbf{x}\|_\infty$. Therefore, by the condition i) of Lemma 1, the following statements are equivalent:

i) The time-varying system (5) is robustly stable.

ii) The set of systems (1) has a CILF $V(\mathbf{x})=\|\mathbf{W}\mathbf{x}\|_\infty$.

To continue, we note that the result of Lemma 2 implies that the set of systems (1) shares a CQLF if and only if the time-varying system (5) is robustly *quadratically* stable, a stricter definition than robust stability given in Definition 3.

Now we suppose that the set of systems (1) has a CQLF $V(\mathbf{x})=\mathbf{x}^T\mathbf{P}\mathbf{x}$, then the time-varying system (5) is robustly quadratically stable. Then by the above equivalent statements i) and ii), the set of systems (1) has a CILF $V(\mathbf{x})=\|\mathbf{W}\mathbf{x}\|_\infty$. This leads to an inclusion relation $\mathcal{L}_Q\subseteq\mathcal{L}_I$.

To show the strictness in the above relation, a numerical example is worked out.

Example 1: Consider a pair of second-order discrete-time systems

$$\mathbf{A}_1=\begin{bmatrix}-79/81 & 0\\ -25/81 & 0\end{bmatrix},\ \mathbf{A}_2=\begin{bmatrix}1/2 & 1/2\\ -1/2 & -1/2\end{bmatrix}. \tag{10}$$

It is easy to see that for the pair of systems, there exists a CILF $V(\mathbf{x})=\|\mathbf{W}\mathbf{x}\|_\infty$, for example, with

$$\mathbf{W}=\begin{bmatrix}1 & 4\\ 3 & 0\end{bmatrix}. \tag{11}$$

Now we check the existence of a CQLF for the pair of the discrete-time systems (10). This is equivalent to the existence of a CQLF for the corresponding pair of continuous-time systems $\{\mathbf{A}_1^*, \mathbf{A}_2^*\}$ obtained via the bilinear transformation

$$\mathbf{A}_i^* = (\mathbf{A}_i + \mathbf{I})^{-1}(\mathbf{A}_i - \mathbf{I}),\ i = 1, 2. \tag{12}$$

This results from the fact that the transformation (12) is known to preserve a CQLF between continuous-time and discrete-time cases [11]. Substituting (10) into (12), we obtain

$$\mathbf{A}_1^* = \begin{bmatrix} -80 & 0 \\ -25 & -1 \end{bmatrix}, \mathbf{A}_2^* = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}. \tag{13}$$

It is shown in [15] that there does not exist a CQLF for the pair of continuous-time systems (13), i.e., there does not exist a CQLF for the pair of discrete-time systems (10). Example 1 shows the inclusion relation between $\mathcal{L}_Q$ and $\mathcal{L}_I$ is really strict, i.e., $\mathcal{L}_Q \subset \mathcal{L}_I$. (QED)

Theorem 1 shows that the strict inclusion relation between the existence conditions of CQLF and CILF holds for the discrete-time case. The continuous-time counterpart has been reported in [15] and thereby we can thus confirm the parallelism between the two cases. The following theorem, besides showing the inclusion relation, further clarifies the relation between matrices $\mathbf{P}$ and $\mathbf{W}$ and validates the corresponding result for the discrete-time case.

Theorem 2 Given the set of systems (1) with a CQLF $V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$, then this set of systems has a CILF $V(\mathbf{x}) = \|\mathbf{W}\mathbf{x}\|_\infty$ with $\mathbf{W} \in \mathcal{R}^{m\times n}$, $m \geq n$, rank$(\mathbf{W}) = n$ defined from the Cholesky factorization $\mathbf{P} = \mathbf{W}^T\mathbf{W}$.

Proof: Suppose that the set of systems (1) has a CQLF $V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$. By using the factorization $\mathbf{P} = \mathbf{W}^T\mathbf{W}$ [5], $\mathbf{W}\in\mathcal{R}^{m\times n}$, $m\geq n$, rank$(\mathbf{W})=n$, we have

$$\begin{aligned} V(\mathbf{x}) &= \mathbf{x}^T\mathbf{P}\mathbf{x} = \mathbf{x}^T\mathbf{W}^T\mathbf{W}\mathbf{x} \\ &= \sum_{i=1}^{m}(\mathbf{w}_i\mathbf{x})^2 = (\|\mathbf{W}\mathbf{x}\|_2)^2. \end{aligned} \tag{14}$$

Here $\mathbf{w}_i$ is the i-th row vector of the matrix $\mathbf{W}$. We now show that $V(\mathbf{x})$ satisfies the condition (9) of Lemma 1. Since the set of systems (1) shares the CQLF $V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$, we have

$$\begin{aligned} &\mathbf{A}^T(t)\mathbf{P}\mathbf{A}(t) - \mathbf{P} < 0,\ \forall\mathbf{A}(t) \in \mathcal{A} \\ &\Leftrightarrow \mathbf{x}^T\mathbf{A}^T(t)\mathbf{P}\mathbf{A}(t)\mathbf{x} - \mathbf{x}^T\mathbf{P}\mathbf{x} < 0,\ \forall\mathbf{A}(t) \in \mathcal{A}, \mathbf{x} \in \mathcal{R}^n \\ &\Leftrightarrow \mathbf{x}^T\mathbf{A}^T(t)\mathbf{P}\mathbf{A}(t)\mathbf{x} < \mathbf{x}^T\mathbf{P}\mathbf{x},\ \forall\mathbf{A}(t) \in \mathcal{A}, \mathbf{x} \in \mathcal{R}^n. \end{aligned} \tag{15}$$

We therefore obtain for any $\mathbf{x} \in \mathcal{R}^n$

$$\begin{aligned} &\max_{\mathbf{y}\in\mathbf{F}(\mathbf{x})} V(\mathbf{y}) = \max_{\mathbf{y}\in\mathbf{F}(\mathbf{x})} \mathbf{y}^T\mathbf{P}\mathbf{y} \\ &= \max_{\mathbf{A}(t)\in\mathcal{A}} \mathbf{x}^T\mathbf{A}^T(t)\mathbf{P}\mathbf{A}(t)\mathbf{x} < \mathbf{x}^T\mathbf{P}\mathbf{x}. \end{aligned} \tag{16}$$

We choose θ satisfying

$$\theta = \max_{\mathbf{y}\in\mathbf{F}(\mathbf{x}),\mathbf{x}\in\mathcal{R}^n,\mathbf{x}\neq 0} \frac{V(\mathbf{y})}{\mathbf{x}^T\mathbf{P}\mathbf{x}}, \tag{17}$$

then $0 < \theta < 1$ and

$$\max_{\mathbf{y}\in\mathbf{F}(\mathbf{x})} V(\mathbf{y}) \leq \theta\mathbf{x}^T\mathbf{P}\mathbf{x} = \theta V(\mathbf{x}). \tag{18}$$

The CQLF function (14) is the Lyapunov function of the form (8) satisfying the condition (9) with $p = 1$. Then by Lemma 1, $V(\mathbf{x}) = \|\mathbf{W}\mathbf{x}\|_\infty$ is a CILF for the set of systems (1), i.e., CQLF and CILF thus defined is connected by $\mathbf{P} = \mathbf{W}^T\mathbf{W}$. (QED)

We now show that for single stable systems as opposed to sets of systems, notwithstanding Theorem 1, the exact equivalent relation holds in the discrete-time case, i.e., the set $\mathcal{L}_Q$ coincides with the set $\mathcal{L}_I$.

Theorem 3 For single discrete-time systems

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t), \mathbf{x} \in \mathcal{R}^n, \mathbf{A} \in \mathcal{R}^{n\times n}, \tag{19}$$

the existence conditions of a quadratic Lyapunov function and of an infinity-norm Lyapunov function are equivalent, and coincide with the Schur stability condition of the matrix $\mathbf{A}$ ($|\lambda(\mathbf{A})| < 1$). Moreover, the relation of matrices $\mathbf{P}$ and $\mathbf{W}$ is given by $\mathbf{P} = \mathbf{W}^T\mathbf{W}$.

Proof: Consider $q = 1$, then the set of systems (1) and the time-varying system (5) become the single *constant* system (19). The trick behind this theorem is that for constant systems, asymptotic stability is identical to quadratic stability. In this case, the conditions i) in Lemma 1 show that the system (19) is asymptotically stable if and only if the system (19) has an infinity-norm Lyapunov function $V(\mathbf{x})=\|\mathbf{W}\mathbf{x}\|_\infty$. On the other hand, the system (19) is asymptotically stable if and only if there exists a quadratic Lyapunov function $V(\mathbf{x})=\mathbf{x}^T\mathbf{P}\mathbf{x}$ or equivalently $|\lambda(\mathbf{A})| < 1$. Therefore, the sets $\mathcal{L}_Q$ and $\mathcal{L}_I$ exactly coincide with each other. Similarly, the relation between matrices $\mathbf{P}$ and $\mathbf{W}$ is given by $\mathbf{P} = \mathbf{W}^T\mathbf{W}$. (QED)

Although the above theorem shows that $\mathcal{L}_Q$ and $\mathcal{L}_I$ are equivalent sets for single stable systems, the form of the matrix $\mathbf{W}$ of CILF may not exist in the square form as the matrix $\mathbf{P}$ of CQLF always does. The following example of a single stable system shows that the CILF with the square form of the matrix $\mathbf{W}$ does not exist.

Example 2: Consider the single stable system with the system matrix

$$\mathbf{A} = \begin{bmatrix} -1/5 & -2/5 \\ 6/5 & -3/5 \end{bmatrix}. \tag{20}$$

We examine the existence of a CILF of the square component matrix

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \tag{21}$$

for this system. By Definition 2, the problem becomes checking the existence of w_{11}, w_{12}, w_{21} and w_{22} such that $\|\mathbf{Q}\|_\infty < 1$, where $\mathbf{Q} = \mathbf{W}\mathbf{A}\mathbf{W}^{-1}$. The checking is done by using the Quantifier Elimination (QE) method, which is a mathematical tool used to solve multivariate polynomial inequalities with quantifiers such as "*for all* ($\forall$)" and "*there exists* ($\exists$)" and quantified variables to obtain symbolic results or logical output such as "*true*" or "*false*" [3], [14]. By applying the QE method, we can confirm the non-existence of a CILF, i.e., there does not exist a CILF of the square matrix $\mathbf{W}$ for this single system.

3.2 Remarks on the CILF Existence Checking

Since common Lyapunov functions can be used in the robust stability analysis and control design for various control systems, it is important to have numerical methods for checking the existence of a common Lyapunov function. As mentioned previously, the existence of a CQLF can be numerically checked by using the available packages such that LMI on the Lyapunov matrix inequalities. Several checkable sufficient conditions for CQLF are also presented in [10], [11]. We have shown that the CILF existence covers a wider class than that of CQLF, but at the cost of unknown number of rows in the component matrix $\mathbf{W}$ (see Example 2). In the case of CILF, however, so far we very few computational methods to check the existence, such as the method to construct infinity-norm Lyapunov functions for a single system in [16], [17]. Therefore, there is a need for testable methods for CILF. Several sufficient existence conditions of a CILF for a set of continuous-time LTI systems are presented in [15]. In the next section, we will present several sufficient existence conditions of a CILF for a set of discrete-time LTI systems that can be applied practically.

3.3 Several Existence Conditions of a CILF

Because of the above-stated reasons, our attention then leads to finding testable methods for checking the existence of a CILF. Theorems 1 and 2 provide help in this regard. In [8], several simple sufficient robust stability conditions for the time-varying system (5) were given. Therefore, these stability conditions give some new sufficient existence conditions of a CILF that can be applied practically. For details, see [8].

Theorem 4 The set of stable systems (1) has a CILF if one of the following conditions is satisfied:

i) The matrices $\mathbf{A}_i, i = 1, ..., q$ are pairwise commutative, i.e., $\mathbf{A}_i\mathbf{A}_j = \mathbf{A}_j\mathbf{A}_i$ for all $i, j = 1, ..., q$.

ii) Every matrix $\mathbf{A}_i, i = 1, ..., q$ is normal. $\mathbf{A} \in \mathcal{R}^{n\times n}$ is said to be normal if $\mathbf{A}^T\mathbf{A} = \mathbf{A}\mathbf{A}^T$.

iii) The matrix $\hat{\mathbf{A}}$ defined below is Schur stable. Given a matrix $\mathbf{A} = [a_{ij}] \in \mathcal{R}^{n\times n}$, we denote $|\mathbf{A}|$ the matrix obtained from $\mathbf{A}$ by taking the absolute value of all entries, i.e., $|\mathbf{A}| = [|a_{ij}|]$, then the nonnegative majorant matrix $\hat{\mathbf{A}}$ is defined by $\hat{\mathbf{A}} = \max_{1\le i\le q}[|\mathbf{A}_i|] = [\max_{1\le i\le q}|a_{ijk}|]$, where the maximum is understood to be entry-wise.

iv) In the given set of matrices $\mathbf{A}_i$, $i = 1, ..., q$, there exists at least one matrix $\mathbf{A}_i$ such that $\mathbf{A}_i = \hat{\mathbf{A}}$ or $\mathbf{A}_i = -\hat{\mathbf{A}}$.

v) In the given set of matrices $\mathbf{A}_i$, $i = 1, ..., q$, there exists at least one matrix $\mathbf{A}_i$ such that $|\mathbf{A}_i| = \hat{\mathbf{A}}$ and $\hat{\mathbf{A}}$ is a Morishima matrix. Here a matrix $\mathbf{A} \in \mathcal{R}^{n\times n}$ is called a Morishima matrix if by symmetric row and column permutations it can be transformed into the form

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix},$$

where $\mathbf{A}_{11} \ge_e 0, \mathbf{A}_{22} \ge_e 0$ are square submatrices and $\mathbf{A}_{12} \le_e 0, \mathbf{A}_{21} \le_e 0$ with $\mathbf{A} \ge_e 0$ ($\mathbf{A} \le_e 0$) denoting that all the elements of a matrix $\mathbf{A}$ are nonnegative (nonpositive).

vi) All the matrices $\mathbf{A}_i$, $i = 1, ..., q$ are either all upper or all lower triangular.

vii) There exists in $\mathcal{R}^n$ a vector norm $\|.\|$ such that the induced matrix norm $\|\mathbf{A}_i\| < 1$ for all $i = 1, ..., q$.

4. Conclusion

In this paper, we have studied the relations between CQLF and CILF for sets of discrete-time LTI systems. It is shown that the exact parallel results between continuous- and discrete-time cases hold. The results indicate that for both continuous- and discrete-time cases, CILF can cover wider classes of systems than CQLF, but with the price of unknown number of rows in the component matrix of CILF. To facilitate the use of CILF, several simple testable existence conditions are provided. Studies on numerical aspects of finding a CILF seem to be still in their infancy and need to be further explored.

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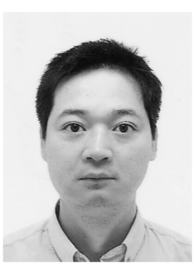

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