IEICE TRANS. ELECTRON., VOL.E90–C, NO.2 FEBRUARY 2007

LETTER

Diffraction Amplitudes from Periodic Neumann Surface: Low Grazing Limit of Incidence (III)

Junichi NAKAYAMA[†a)], **Kazuhiro HATTORI**[†], *and* **Yasuhiko TAMURA**[†], *Members*

SUMMARY This paper deals with the singular behavior of the diffraction of transverse magnetic (TM) waves by a perfectly conductive triangular periodic surface at a low grazing limit of incidence. The wave field above the highest excursion of the surface is represented as a sum of Floquet modes with modified diffraction amplitudes, whereas the wave field inside a triangular groove is written as a sum of guided modes with unknown mode amplitudes. Then, two sets of equations are derived for such amplitudes. From the equation sets, all the amplitudes are analytically shown to vanish at a low grazing limit of incidence. From this fact, it is concluded analytically that no diffraction takes place and only reflection occurs at a low grazing limit of incidence for any period length and any triangle height. This theoretical result is verified by a numerical example.

key words: *singular behavior, diffraction amplitude, modified diffraction amplitude, triangular grating*

1. Introduction

This paper deals with TM plane wave diffraction by a perfectly conductive periodic surface at a low grazing limit of incidence. This problem is of practical interest for ground-based radar studies of the sea surface and for long-path propagation along the sea surface [1].

It is known that scattering or diffraction by a rough Neumann surface does not take place and only reflection appears at a low grazing limit of incidence (LGLI). Such a singular behavior was predicted for a homogeneous random surface [2]–[4] and a periodic rough surface [5], [6], but discussions were all restricted to a slightly rough case.

Recently, however, we predicted [6] that such a singular behavior is generally true even for a very rough periodic Neumann surface. By the modal expansion method [7], we demonstrated that our prediction holds for a periodic array of rectangular grooves with a deep groove depth [8].

In this paper, we present another example, which is diffraction by a periodic array of symmetric triangular grooves, as shown in Fig. 1. By making use of the modal expansion method and the concept of the modified diffraction amplitude [6], we find analytically that the diffraction amplitudes vanish but the reflection coefficient becomes -1 at LGLI. This analytical result is verified by a numerical example.

Manuscript received August 21, 2006.
Manuscript revised October 12, 2006.

[†]The authors are with the Graduate School of Engineering and Design, Kyoto Institute of Technology, Kyoto-shi, 606-8585 Japan.
a) E-mail: nakayama@kit.ac.jp

DOI: 10.1093/ietele/e90–c.2.536

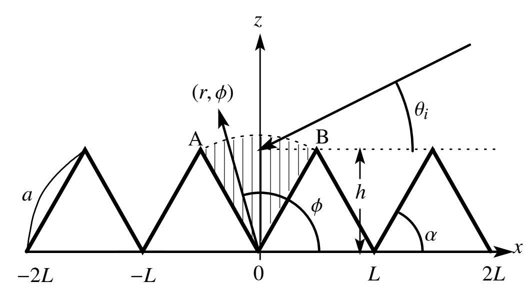

Fig. 1 Diffraction by a symmetric triangular periodic surface. L is the period, h is the altitude, and α is the base angle.

2. Modal Expansion Method

Let us consider a triangular grating with period L and altitude h (see Fig. 1.):

$$L = 2a\cos\alpha, \quad h = a\sin\alpha = \frac{L}{2}\tan\alpha, \tag{1}$$

$$a = \sqrt{h^2 + L^2/4}, \tag{2}$$

where α is the base angle and a is the side length or the radius of the hatched sector. We introduce a cylindrical coordinate system (r, ϕ) as

$$x = r\cos\phi, \quad z = r\sin\phi. \tag{3}$$

We write the y component of the magnetic field by $\psi(x, z)$, which satisfies

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right]\psi(x, z) = 0 \tag{4}$$

in the region above the surface and the Neumann condition on the triangular surface,

$$\frac{\partial}{\partial n}\psi(x, z) = 0. \tag{5}$$

Here, $k = 2\pi/\lambda$ is wave number. However, we denote $\psi(x, z)$ in region $z \geq h$ by $\psi_1(x, z)$. Due to the periodic nature of the problem, $\psi_1(x, z)$ may have the Floquet form, which we represent as

$$\psi_1(x, z) = e^{-ipx-i\beta_0(z-h)} - e^{-ipx+i\beta_0(z-h)} + \sum_{n=-\infty}^{\infty}(A_n + \delta_{n,0})e^{-i(p+mk_L)x+i\beta_m(z-h)}. \tag{6}$$

Here p, β_n and k_L are defined by

$$p = k\cos\theta_i, \quad \beta_0 = k\sin\theta_i, \tag{7}$$

$$\beta_n = \sqrt{k^2 - (p + nk_L)^2}, \quad (n = 0, \pm 1, \pm 2, \cdots), \tag{8}$$

$$Re[\beta_n] \geq 0, \quad Im[\beta_n] \geq 0, \tag{9}$$

$$k_L = 2\pi/L, \tag{10}$$

where θ_i is the angle of incidence measured from the positive x axis (see Fig. 1), Re and Im are the real and imaginary parts, respectively, and k_L is the spatial angular frequency of period L. The first term on the right-hand side of (6) is the incident plane wave, the second term is the reflected wave with the amplitude -1, and the summation is the modified diffracted wave. A_n is the nth-order diffraction amplitude and A_0 is the reflection coefficient. However, $(A_n + \delta_{n,0})$ is the nth-order modified diffraction amplitude [6], which is the key idea in discussions below. Note that $\beta_0 = k\sin\theta_i \to 0$ when $\theta_i \to 0$ or π.

On the other hand, we denote by $\psi_2(x,z)$ the y component of the magnetic field in the hatched sector $(0 \leq r \leq a, \alpha \leq \phi \leq \pi - \alpha)$ in Fig. 1. In such a sector, we write $\psi_2(x,z)$ as a sum of radial line modes [7],

$$\psi_2(x,z) = \sum_{n=0}^{\infty} D_n J_{\nu(n)}(kr)\cos\left[\nu(n)(\theta - \alpha)\right], \tag{11}$$

$$\nu(n) = n\pi/(\pi - 2\alpha), \tag{12}$$

where $J_{\nu(n)}$ is Bessel function with the order $\nu(n)$.

Since ψ and $\partial\psi/\partial r$ are continuous across the arc AB of the hatched sector, $(r = a, \alpha \leq \phi \leq \pi - \alpha)$, in Fig. 1, we obtain

$$\psi_1(a\cos\phi, a\sin\phi) - \psi_2(a\cos\phi, a\sin\phi) = 0, \tag{13}$$

$$\frac{\partial}{\partial r}\left[\psi_1(r\cos\phi, r\sin\phi) - \psi_2(r\cos\phi, r\sin\phi)\right]\Big|_{r=a} = 0. \tag{14}$$

We multiply (13) and (14) by $\cos[\nu(m)(\phi - \alpha)]$, and integrate the results with respect to ϕ over the interval $[\alpha, \pi - \alpha]$. Using

$$\frac{dJ_\nu(r)}{dr} = \frac{\nu}{r}J_\nu(r) - J_{\nu+1}(r), \tag{15}$$

$$\int_\alpha^{\pi-\alpha} \cos[\nu(m)(\phi - \alpha)]\cos[\nu(n)(\phi - \alpha)]d\phi = \frac{\pi - 2\alpha}{2}\delta_{m,n}(\delta_{m,0} + 1), \tag{16}$$

$\delta_{m,n}$ being Kronecker's delta, we obtain, after some manipulation, the linear equation systems for the vector $[\{A_n + \delta_{n,0}\}, \{D_m\}]$ as

$$\sum_{n=-\infty}^{\infty} c_{mn}^{(11)}(A_n + \delta_{n,0}) - c_{mm}^{(12)}D_m = \beta_0 e_1(m), \tag{17}$$

$$\sum_{n=-\infty}^{\infty} c_{mn}^{(21)}(A_n + \delta_{n,0}) - c_{mm}^{(22)}D_m = \beta_0 e_2(m), \tag{18}$$

which hold for $m = 0, 1, 2, \cdots$. Here, we have

$$c_{mn}^{(11)} = \int_\alpha^{\pi-\alpha} \cos[\nu(m)(\phi - \alpha)]e^{-i(p+nk_L)a\cos\phi} \times e^{i\beta_n(a\sin\phi - h)}d\phi, \tag{19}$$

$$c_{mn}^{(12)} = \delta_{mn}\frac{\pi - 2\alpha}{2}(1 + \delta_{m,0})J_{\nu(m)}(ka), \tag{20}$$

$$c_{mn}^{(21)} = \int_\alpha^{\pi-\alpha} \cos[\nu(m)(\phi - \alpha)][-i(p + nk_L)\cos\phi + i\beta_n\sin\phi]e^{-i(p+nk_L)a\cos\phi + i\beta_n(a\sin\phi - h)}d\phi, \tag{21}$$

$$c_{mn}^{(22)} = \delta_{mn}\left[\frac{\nu(m)}{a}J_{\nu(m)}(ka) - kJ_{\nu(m)+1}(ka)\right] \times \frac{\pi - 2\alpha}{2}(1 + \delta_{m,0}), \tag{22}$$

$$e_1(m) = 2i\int_\alpha^{\pi-\alpha} \cos[\nu(m)(\phi - \alpha)]e^{-ipa\cos\phi} \times \frac{\sin[\beta_0(a\sin\phi - h)]}{\beta_0}d\phi, \tag{23}$$

$$e_2(m) = 2\int_\alpha^{\pi-\alpha} \cos[\nu(m)(\phi - \alpha)]e^{-ipa\cos\phi}\Big[p\cos\phi \times \frac{\sin[\beta_0(a\sin\phi - h)]}{\beta_0} + i\sin\phi\cos[\beta_0(a\sin\phi - h)]\Big]d\phi. \tag{24}$$

Since $(a\sin\phi - h) \geq 0$ holds for $\alpha \leq \phi \leq \pi - \alpha$, the integrals in (19) and (21) are finite for any m and n. Also, $c_{mn}^{(12)}$ and $c_{mn}^{(22)}$ are finite. Furthermore, $e_1(m)$ and $e_2(m)$ remain finite for any $\beta_0 \geq 0$, and hence, the right-hand sides of the systems (17) and (18) vanish when $\beta_0 \to 0$. However, it is difficult to prove the unique existence of the vector solution $[\{A_n + \delta_{n,0}\}, \{D_m\}]$[†]. In what follows, however, we simply assume the existence of the unique solution. This means that the vector solution vanishes if the right-hand sides of the systems become 0. Since the right-hand sides of (17) and (18) have the common factor β_0 and vanish when $\beta_0 \to 0$, the vector solution has the factor β_0 and vanishes when $\beta_0 \to 0$. Therefore, we may write

$$A_n + \delta_{n,0} = \beta_0 a_n \to 0, \quad (\beta_0 \to 0), \tag{25}$$

$$D_m = \beta_0 d_m \to 0, \quad (\beta_0 \to 0). \tag{26}$$

Here, a_n and d_m depend on λ, p, L and h. Rewriting (25), we obtain an important result,

$$\lim_{\beta_0 \to 0} A_0 = \lim_{\beta_0 \to 0}(\beta_0 a_0 - 1) = -1,$$

$$\lim_{\beta_0 \to 0} A_n = \lim_{\beta_0 \to 0}(\beta_0 a_n) = 0, \quad (n \neq 0), \tag{27}$$

which means that only reflection appears at LGLI. This also means that diffraction amplitude A_n $(n \neq 0)$ decreases in proportion to β_0 and vanishes at LGLI. We note that such a singularity holds for any length of period L and for any altitude h in the analytical sense[††].

[†]It is pointed out that no uniqueness proof exists for wave diffraction by a periodic Neumann surface [9].

[††]In the numerical sense, this is not always true. By use of the truncation (28), a highly accurate numerical solution is easily obtained from (17) and (18) if L and h are not very large but becomes difficult to obtain when L is much larger than λ.

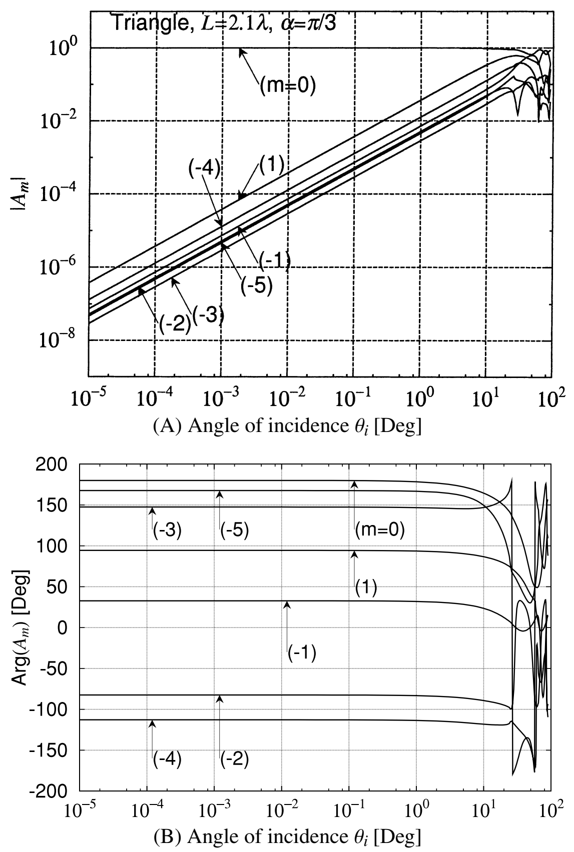

Fig. 2 (A) Amplitude and (B) phase of the diffraction amplitudes. $L = 2.1\lambda$, $\theta_s = \pi/3$, $N_F = 18$.

3. Numerical Example and Conclusion

For numerical analysis, we assume

$$\begin{aligned} &A_n + \delta_{n,0} = 0, \quad |n| > N_F, \\ &D_m = 0, \qquad\quad\; m > 2N_F, \end{aligned} \tag{28}$$

where N_F is the truncation number. By the truncation (28), (17) and (18) become $(2N_F + 1)$-equations which determine the $2(2N_F + 1)$-vector $[\{A_{-N_F}, A_{1-N_F}, \cdots, A_{N_F}\}, \{D_0, D_1, \cdots, D_{2N_F}\}]$. Setting $N_F = 18$ and

$$L = 2.1\lambda, \quad \alpha = \pi/3, \quad h \approx 1.8186\lambda, \tag{29}$$

we solved such truncated (17) and (18) numerically for the angle of incidence θ_i from 0.0001° to 90°. In the case of (29), the energy error is less than 10^{-3} for any angle of incidence and it becomes less than 10^{-6} when $\theta_i \le 0.1°$. The result is illustrated in Fig. 2. Figure 2(A) shows that the diffraction amplitude $|A_n|$ $(n \neq 0)$ decreases in proportion to $\beta_0 = k \sin\theta_i \approx k\theta_i$, and the reflection coefficient $|A_0|$ is almost equal to 1 when $\theta_i \le 1°$, as is expected theoretically. Figure 2(B) shows that reflection phase $Arg(A_0)$ is almost equal to 180° when $\theta_i \le 0.1°$. This numerical example supports our theoretical conclusion (27) even when triangle altitude h is larger than wavelength λ.

The scattering or diffraction from a rough Neumann surface does not take place and only reflection appears at a low grazing limit of incidence. Such a singular behavior was known in the case of a slightly rough Neumann surface. In this paper, however, we newly demonstrated analytically and numerically that such a singular behavior takes place for a symmetric triangular periodic grating with large altitude h. From this and the result in [8], we may conclude that our prediction must hold for any periodic Neumann surface.

It is important to note that (27) and (26) cause $\psi_1(x, z)$ in (6) and $\psi_2(x, z)$ in (11) to vanish at LGLI. Therefore, the region above the surface becomes *shadow*. Such a shadow takes place at LGLI, because the incident plane wave is completely canceled by the reflected wave with amplitude -1 (the second term in (6)). The physical processes that yield such a shadow are not clear. This problem is left for a future study.

References

[1] D.E. Barrick, "Grazing behavior of scatter and propagation above any rough surface," IEEE Trans. Antennas Propag., vol.46, no.1, pp.73–83, 1998.
[2] F.G. Bass and I.M. Fuks, Wave Scattering from Statistically Rough Surfaces, Pergamon Press, New York, 1979.
[3] J. Nakayama, "Anomalous scattering from a slightly random surface," Radio Sci., vol.17, no.3, pp.558–564, 1982.
[4] Y. Tamura and J. Nakayama, "Mass operator for wave scattering from a slightly random surface," Waves in Random Media, vol.9, pp.341–368, 1999.
[5] M.I. Charnotskii, "Wave scattering by periodic surface at low grazing angles: Single grazing mode," Progress in Electromagnetic Research (PIER) 26, pp.1–42, 2000.
[6] J. Nakayama, K. Hattori, and Y. Tamura, "Diffraction amplitudes from periodic Neumann surface: Low grazing limit of incidence," IEICE Trans. Electron., vol.E89-C, no.5, pp.642–644, May 2006.
[7] R. Petit, ed. Electromagnetic theory of gratings, Springer, Berlin, 1980.
[8] J. Nakayama, K. Hattori, and Y. Tamura, "Diffraction amplitudes from periodic Neumann surface: Low grazing limit of incidence (II)," IEICE Trans. Electron., vol.E89-C, no.9, pp.1362–1364, Sept. 2006.
[9] J.A. DeSanto, "Scattering from a perfectly reflecting arbitrary periodic surface: An exact theory," Radio Sci., vol.16, no.6, pp.1315–1326, 1981.