PAPER

Wave Scattering from a Finite Periodic Surface: Spectral Formalism for TE Wave

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SUMMARY This paper deals with the wave scattering from a periodic surface with finite extent. Modifying a spectral formalism, we find that the spectral amplitude of the scattered wave can be determined by the surface field on only the corrugated part of the surface. The surface field on such a corrugated part is then expanded into Fourier series with unknown Fourier coefficients. A matrix equation for the Fourier coefficients is obtained and is solved numerically for a sinusoidally corrugated surface. Then, the angular distribution of the scattering, the relative power of each diffraction beam and the optical theorem are calculated and illustrated in figures. Also, the relative powers of diffraction are calculated against the angle of incidence for a periodic surface with infinite extent. By comparing a finite periodic case with an infinite periodic case, it is pointed out that relative powers of diffraction beam are much similar in these of diffraction for the infinite periodic case.

key words: *wave scattering, finite periodic surface, diffraction beam, non-Rayleigh approach*

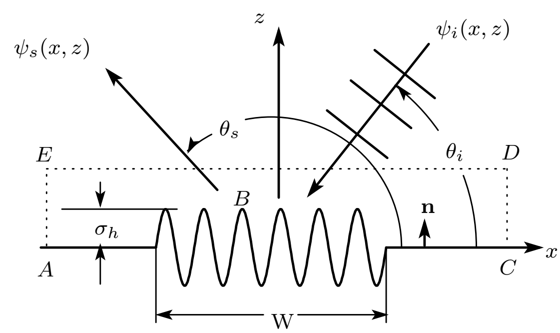

Fig. 1 Scattering of a plane wave from a periodic surface. θ_i is the angle of incidence and θ_s is a scattering angle. W is the width of periodic corrugation.

1. Introduction

This paper deals with the scattering from a periodically corrugated surface with finite extent (See Fig. 1). Such a problem was studied by Maystre [1] by an integral equation method, where the surface field on the entire surface is determined in the coordinate domain. On the other hand, we introduced the periodic Fourier transform [2] and the diffraction beam [3] as new concepts of analysis. In case of a sinusoidal surface with finite extent we presented a wave solution to calculate the angular distribution of the scattering and the optical theorem. However, the solution is based on the Rayleigh hypothesis and hence is applicable only when the surface deformation is small compared with the wavelength.

This paper deals with a non-Rayleigh approach based on the the spectral formalism [4], [5], where the surface field is determined by an integral equation and the spectral amplitude of the scattered wave is calculated from the surface field on the entire surface. Modifying the spectral formalism, however, we present a formulation suitable for the scattering from a finite periodic surface. We first point out a fact that the spectral amplitude can be obtained only by the surface field on the corrugated part of the surface. Taking this fact, we express the surface field on such a corrugation with finite extent into Fourier series with unknown Fourier coefficients. Then, the integral equation for the surface field is reduced to a matrix equation for the Fourier coefficients. We solve the matrix equation numerically for a sinusoidally corrugated surface to obtain the spectral amplitude of the scattered wave, from which the angular distribution of the scattering is calculated. We also calculate the optical theorem and the relative power of diffraction beam.

We have been looking for a relation between the scattering from a finite periodic surface and the diffraction by a periodic surface with infinite extent [2], [3], [6]. We have proposed an expectation such that the relative powers of diffraction beams are much similar to the relative powers of diffraction by an infinite periodic surface [3]. In terms of numerical results, we briefly discuss the validity of such expectation.

2. Diffraction Beam and Optical Theorem

This section describes a mathematical formulation of the problem. A brief description is given on the extended Floquet form, the diffraction beam and the relative power of diffraction beam, which were introduced previously [2], [3].

Let us consider the wave scattering from a finite periodic plane. We write the surface corrugation as

$$z = f(x) = u(x|W)f_p(x), \tag{1}$$

where $f_p(x)$ is a periodic function with the period L,

$$f_p(x) = f_p(x+L), \tag{2}$$

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and $u(x|W)$ is the rectangular pulse given by

$$u(x|W) = \begin{cases} 0, & |x| > W/2 \\ 1, & |x| < W/2 \end{cases}. \tag{3}$$

For convenience, we put

$$k_L = \frac{2\pi}{L}, \quad k_W = \frac{2\pi}{W}. \tag{4}$$

We denote the y component of the electric field by $\psi(x, z)$, which satisfies Helmholtz equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right] \psi(x, z) = 0, \tag{5}$$

in the region $z > f(x)$ and the Dirichlet condition

$$\psi(x, z)|_{z=f(x)} = 0, \tag{6}$$

on the surface $z = f(x)$. Here, $k = 2\pi/\lambda$ is wavenumber and λ is the wavelength. We write the incident plane wave $\psi_i(x, z)$ as

$$\psi_i(x, z) = e^{-ipx} e^{-i\beta_0(p)z}, \quad p = k \cdot \cos\theta_i, \tag{7}$$

$$\beta_m(p) = \beta_0(p + mk_L) = \sqrt{k^2 - (p + mk_L)^2},$$
$$Im\left[\beta_m(p)\right] \geqq 0, \quad (m = 0, \pm 1, \pm 2, \cdots), \tag{8}$$

where θ_i is the angle of incidence and Im stands for imaginary part. Since the surface is flat for $|x| > W/2$, we put the y component of the electric field as

$$\psi(x, z) = e^{-ipx}[e^{-i\beta_0(p)z} - e^{i\beta_0(p)z}] + \psi_s(x, z), \tag{9}$$

which is a sum of incident plane wave, the specularly reflected wave and $\psi_s(x, z)$ the scattered wave due to surface deformation. By use of the periodic Fourier transform [3], it was shown that the scattered wave field has an extended Floquet form, which we write as

$$\psi_s(x, z) = \sum_{m=-\infty}^{\infty} \psi_m(x, z). \tag{10}$$

$$= \int_{-\infty}^{\infty} A(s) e^{-i(p+s)x + i\beta_0(p+s)z} ds, \tag{11}$$

where (11) and (10) hold for $z > \max\{f(x)\}$. Equation (11) is a Fourier spectrum representation and $A(s)$ is the spectral amplitude, However, (10) is the extended Floquet form, where $\psi_m(x, z)$ is the m-th order diffraction beam [3] given by

$$\psi_m(x, z) = \frac{1}{k_L} \int_{-\pi/L}^{\pi/L} A_m(s) e^{-i(p+s+mk_L)x} \times e^{i\beta_m(p+s)z} ds. \tag{12}$$

Here, $A_m(s)$ is related with the spectral amplitude $A(s)$ as

$$A_m(s) = k_L \cdot A(s + mk_L) u(s|k_L), \tag{13}$$

where $u(s|k_L)$ is the function defined by (3). The diffraction beams are orthogonal in the sense that

$$Re\left(\int_{-\infty}^{\infty} \frac{\partial \Psi_m(x, z)}{i\partial z} \Psi_n^*(x, z) dx\right) = \delta_{mn} \Phi_m, \tag{14}$$

where δ_{mn} is Kronecker's delta, the asterisk and Re stand for complex conjugate and real part, respectively. Φ_m is the power carried by the m-th order diffraction beam into the z direction,

$$\Phi_m = 2\pi \int_{-k_L/2}^{k_L/2} Re\left[\frac{\beta_m(p+s)}{k_L^2}\right] |A_m(s)|^2 ds, \tag{15}$$

The optical theorem may be given by

$$\frac{4\pi}{k_L} \beta_0(p) Re[A_0(0)] = \frac{kW}{2\pi} \int_0^{\pi} \sigma(\theta_s|\theta_i) d\theta_s$$
$$= \sum_{n=-\infty}^{\infty} \Phi_m, \tag{16}$$

where $4\pi\beta_0(p)Re[A_0(0)]/k_L$ is the total power of scattering, and $\sigma(\theta_s|\theta_i)$ is the differential scattering cross section divided by W,

$$\sigma(\theta_s|\theta_i) = \frac{(2\pi)^2 k}{W} \sin^2\theta_s |A(-k\cos\theta_s - p)|^2, \tag{17}$$

where θ_s is a scattering angle (See Fig. 1). The relation (16) means that the scattering takes place with the loss of the specularly scattering component.

Assuming $\beta_0(p) = k\sin(\theta_i) \neq 0$, we rewrite the optical theorem as

$$\sum_{m=-\infty}^{\infty} P_m = 1, \quad P_m = \frac{k_L \cdot \Phi_m}{4\pi\beta_0(p) Re[A_0(0)]}, \tag{18}$$

where P_m is the relative power of the m-th order diffraction beam divided by the total scattering power $4\pi\beta_0(p)Re[A_0(0)]/k_L$. We will use (18) to check accuracy of numerical calculation below.

Roughly speaking, the total power of scattering and the power Φ_m are proportional to W, because the scattering takes place at the corrugated part of the surface. Thus, the relative power P_m is expected to become almost independent of W. This implies an expectation [3] such that the relative powers of diffraction beams are much similar to the diffraction powers for an infinite periodic surface with $W \to \infty$. We will discuss such an expectation below.

3. Equation in Spectral Domain

In the spectral formalism by DeSanto [4], [5], auxiliary functions were up-going and incoming plane waves. Modifying them, however, we put auxiliary functions as

$$G^{(1)}(p + s, x, z) = e^{i(p+s)x + i\beta_0(p+s)z}, \tag{19}$$

$$G^{(2)}(p + s, x, z)$$

$$= e^{i(p+s)x}\left[e^{i\beta_0(p+s)z} - e^{-i\beta_0(p+s)z}\right], \quad (20)$$

which satisfy Helmholtz equation. Here, $G^{(1)}(p+s,x,z)$ is an up-going plane wave and $G^{(2)}(p+s,x,z)$ is a standing wave vanishing at $z=0$. Integrating the identity $\mathrm{div}[G^{(j)}\cdot\mathrm{grad}\psi - \psi\cdot\mathrm{grad}G^{(j)}] = 0$, $(j=1,\ 2)$, over a box area $ABCDEA$ in Fig. 1 and applying Gauss's theorem, one easily finds

$$\int_{ABCDEA}\left(G^{(j)}\frac{\partial\psi}{\partial n} - \psi\frac{\partial G^{(j)}}{\partial n}\right)dl = 0, \quad (21)$$

where $j=1,\ 2$. Applying the boundary condition (6) and taking a limit $C,D\to\infty$ and $A,E\to-\infty$, we obtain

$$\lim_{\substack{D\to\infty\\E\to-\infty}}\int_{ED}\left[G^{(j)}\frac{\partial\psi}{\partial z} - \psi\frac{\partial G^{(j)}}{\partial z}\right]dx$$
$$= \lim_{\substack{A\to-\infty\\B\to\infty}}\int_{ABC}\frac{\partial\psi}{\partial n}G^{(j)}dl, \quad (j=1,\ 2), \quad (22)$$

where $dl=\sqrt{1+(df/dx)^2}dx$ is the arc length along the surface†.

Substituting (9) and (20) into (22), we obtain a relation determining $A(s)$ from the surface field $\partial\psi/\partial n$,

$$\int_{-W/2}^{W/2}\left[e^{i\beta_0(p+s)f(x)} - e^{-i\beta_0(p+s)f(x)}\right]$$
$$\times e^{i(p+s)x}\frac{\partial\psi}{\partial n}\frac{dl}{dx}dx = -4\pi i\beta_0(p+s)A(s), \quad (23)$$

which is the key equation in this paper. The relation (23) differs from Maystre [1] and DeSanto [5], in which the scattered wave is obtained from the surface field on the entire surface. However, the left hand side of (23) is an integral over $[-W/2, W/2]$. This means that there is no need to determine the surface field $\partial\psi/\partial n$ on the entire surface. To determine the spectral amplitude $A(s)$, we only need the surface field on the corrugated portion of the surface. Taking this advantage of our formulation, we will obtain an equation determining $\partial\psi/\partial n$ for $|x|<W/2$.

On the other hand, one finds from (19) and (22),

$$\int_{-W/2}^{W/2}\left[e^{i\beta_0(p+s)f(x)} - 1\right]e^{i(p+s)x}\frac{\partial\psi}{\partial n}\frac{dl}{dx}dx$$
$$+\int_{-\infty}^{\infty}e^{i(p+s)x}\frac{\partial\psi}{\partial n}\frac{dl}{dx}dx = -4\pi i\beta_0(p+s)\delta(s) \quad (24)$$

which is an integral equation determining $\partial\psi/\partial n$ over the entire surface and $\delta(s)$ is Dirac's delta function. We put the surface field as

$$\frac{\partial\psi}{\partial n} = \frac{e^{-ipx-i\beta_0(p)f(x)}}{\sqrt{1+(df/dx)^2}}[-2i\beta_0(p)+v_B(x)], \quad (25)$$

where $-2i\beta_0(p)e^{-ipx-i\beta_0(p)f(x)}/\sqrt{1+(df/dx)^2}$ is the surface field obtained by the tangential plane approximation. To obtain the surface field $\partial\psi/\partial n$, we assume that $v_B(x)$ is well approximated by a band-limited function and we put

$$v_B(x) = \int_{-k_B}^{k_B}B(s',k_B)e^{-is'x}ds', \quad (26)$$

where $B(s',k_B)$ is a function to be determined Here, $B(s,k_B)=0$ for $|s|>k_B$ is assumed, which should be understood as truncation. From the discussion above, we only need $v_B(x)$ for $|x|<W/2$. Therefore, we expand $v_B(x)$ into Fourier series as

$$v_B(x) = \frac{1}{W}\sum_{m=-\infty}^{\infty}B_m e^{-imk_Wx}, \quad |x|<W/2, \quad (27)$$

$$B_m = \int_{-k_B}^{k_B}U(s'-mk_W|W)B(s',k_B)ds', \quad (28)$$

where $U(s|W)$ is the Fourier transform of $u(x|W)$

$$U(s|W) = \int_{-W/2}^{W/2}e^{isx}dx = \frac{\sin(Ws/2)}{(s/2)}. \quad (29)$$

Let us obtain an equation for B_m from (24). We first expand $e^{isx}(e^{i\beta f_p(x)}-1)$ into a Fourier series with the period W,

$$e^{isx}\left[e^{i\beta f_p(x)}-1\right]$$
$$= \sum_{n=-\infty}^{\infty}C_n(s|\beta)e^{ink_Wx}, \quad |x|<W/2, \quad (30)$$

$$C_n(s|\beta) = \frac{1}{W}\int_{-W/2}^{W/2}\left[e^{i\beta f_p(x)}-1\right]e^{i(s-nK_W)x}dx. \quad (31)$$

Then, we substitute (26), (25), and (30) into (24) to obtain,

$$2\pi B(s,k_B) - 2iW\beta_0(p)C_0(s|\beta_0(p+s)-\beta_0(p))$$
$$+\sum_{n=-\infty}^{\infty}C_n(s|\beta(p+s)-\beta_0(p))B_n = 0, \quad (32)$$

which together with (28) is regarded as an integral equation in the spectral domain. Multiplying $U(s-mk_W|W)$ to the both sides of (32) and integrating the result over s region with $[-k_B,k_B]$, however, we obtain a matrix equation for B_m,

$$B_m + \sum_{n=-\infty}^{\infty}D_{mn}B_n = E_m, \quad (33)$$

where we have put

$$D_{mn} = \frac{1}{2\pi}\int_{-k_B}^{k_B}C_n(s|\beta_0(p+s)-\beta_0(p))$$

†Here, the integrals over CD and EA are neglected [5].

$$\times U(s - mk_W|W)ds, \tag{34}$$

$$E_m = 2iW\beta_0(p)D_{m0}. \tag{35}$$

Next, let us obtain another equation to calculate $A(s)$ from B_m. Substituting (26), (25), and (30) into (23), we obtain

$$\begin{aligned}4\pi i\beta_0(p+s)A(s) &= 2iW\beta_0(p)[C_0(s|\beta_0(p+s)\\ &-\beta_0(p)) - C_0(s|-\beta_0(p+s)-\beta_0(p))]\\ &+\sum_{n=-\infty}^{\infty}[C_n(s|-\beta_0(p+s)-\beta_0(p))\\ &-C_n(s|\beta_0(p+s)-\beta_0(p))]B_n.\end{aligned} \tag{36}$$

Dividing the both sides of this by $\beta_0(p+s)$, we finally get an equation to calculate $A(s)$ from B_n,

$$\begin{aligned}A(s) \;=\; &\frac{W\beta_0(p)}{2\pi}C_0'(s|\beta_0(p+s),\beta_0(p))\\ &-\frac{1}{4\pi i}\sum_{n=-\infty}^{\infty}C_n'(s|\beta_0(p+s),\beta_0(p))B_n,\end{aligned} \tag{37}$$

where we have put

$$C_n'(s|\beta,\gamma) = \frac{C_n(s|\beta-\gamma)-C_n(s|-\beta-\gamma)}{\beta}. \tag{38}$$

This remains finite even when $\beta = 0$. Since $\beta(p+s) = k\sin(\theta_s)$, θ_s being a scattering angle, we find from (17), (37) and (38)

$$\lim_{\theta_s\to 0,\ \pi}\sigma(\theta_s|\theta_i) = 0. \tag{39}$$

Thus, the scattering cross section for TE case vanishes when the scattering angle is grazing.

4. Numerical Example for Sinusoidal Case

Let us consider a simple example, where the surface deformation is sinusoidal

$$f_p(x) = \sigma_h \sin(k_L x). \tag{40}$$

To make $f(x)$ a continuous function of x, we put

$$W = N_{WL}\cdot L, \tag{41}$$

where N_{WL} is implicitly assumed to be an integer. By use of formulas on the Bessel function $J_l(\cdot)$,

$$e^{i\sigma_h\beta\sin(k_Lx)} = \sum_{l=-\infty}^{\infty}J_l(\sigma_h\beta)e^{ilk_Lx}, \tag{42}$$

we obtain from (31)

$$\begin{aligned}C_n(s|\beta) = \frac{1}{W}&\sum_{l=-\infty}^{\infty}[J_l(\sigma_h\beta)-\delta_{l0}]\\ &\times U(s+lk_L-nk_W|W).\end{aligned} \tag{43}$$

Substituting this into (34), we obtain a matrix element

$$\begin{aligned}D_{mn} = \frac{1}{2\pi W}&\int_{-k_B}^{k_B}\sum_{l=-\infty}^{\infty}\{J_l(\sigma_h[\beta_0(p+s)-\beta_0(p)])\\ &-\delta_{l0}\}\cdot U(s+lk_L-nk_W|W)U(s-mk_W|W)ds.\end{aligned} \tag{44}$$

By (31) and (38) we obtain two expressions for $C_n'(s|\beta,\gamma)$,

$$\begin{aligned}C_n'(s|\beta,\gamma) &= \frac{1}{W}\int_{-W/2}^{W/2}\Big(e^{i\sigma_h(\beta-\gamma)\sin(k_Lx)}\\ &\quad -e^{-i\sigma_h(\beta+\gamma)\sin(k_Lx)}\Big)\frac{e^{i(s-nk_W)x}}{\beta}dx \quad (45)\\ &= \frac{2i}{W}\int_{-W/2}^{W/2}e^{-i\sigma_h\gamma\sin(k_Lx)}\\ &\quad\times\frac{\sin(\sigma_h\beta\sin(k_Lx))}{\beta}e^{i(s-nk_W)x}dx. \quad (46)\end{aligned}$$

When $\beta \neq 0$, we obtain from (45) and (42),

$$\begin{aligned}C_n'(s|\beta,\gamma) = \frac{1}{W}&\sum_{l=-\infty}^{\infty}\frac{U(s+lk_L-nk_W|W)}{\beta}\\ &\times\{J_l(\sigma_h(\beta-\gamma))-J_l(-\sigma_h(\beta+\gamma))\}.\end{aligned} \tag{47}$$

When $\beta = 0$, however, we calculate (46). Since $\sin(\sigma_h\beta\sin(k_Lx))/\beta \to \sigma_h\sin(k_Lx)$ when $\beta\to 0$, we obtain from (46) and (42),

$$\begin{aligned}\lim_{\beta\to 0}C_n'(s|\beta,\gamma) = \frac{\sigma_h}{W}&\sum_{l=-\infty}^{\infty}J_l(-\sigma_h\gamma)[U(s-nk_W\\ &+(l+1)k_L|W)-U(s-nk_W+(l-1)k_L|W)].\end{aligned} \tag{48}$$

We will use (47) and (48) in numerical calculations below.

For numerical calculation, we put

$$L = 2.5\lambda,\quad N_{WL} = 20,\quad W = N_{WL}L = 20L. \tag{49}$$

Introducing the truncation number N_M of diffraction beams, we set

$$N_M = 8,\quad N_B = N_MN_{WL} = 160. \tag{50}$$

This means that the summation in (33) is approximated by a finite sum from $n = -N_B$ to $N_B = 160$. Thus, $\{B_m\}$ becomes a $(2N_B+1)$-vector and $[D_{mn}]$ is a $(2N_B+1)\times(2N_B+1)$ matrix in the calculation below. To calculate the matrix element in (44), we put the band width parameter k_B as

$$k_B = (N_M+0.5)N_{WL}k_W = (N_M+0.5)k_L. \tag{51}$$

We numerically calculated D_{mn} in (44) and then solve (33) for B_m. Then we apply (37) and (13) to obtain $A(s)$ and $A_m(s)$, from which the relative power P_m, the scattering cross section $\sigma(\theta_s|\theta_i)$ and optical theorem (18) are calculated numerically.

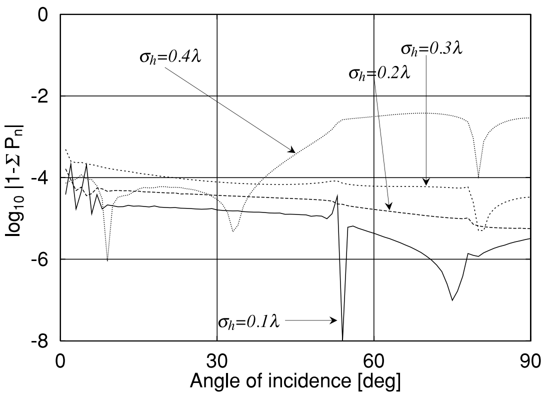

Fig. 2 Log-Error on optical theorem. Period $L = 2.5\lambda$, width $W = 50\lambda$. σ_h is the surface height and λ is wavelength. $N_B = 160$, $k_B/k_L = 8.5$.

Figure 2 illustrates the error $|1 - \sum_n P_n|$ against the angle of incidence θ_i for $\sigma_h = 0.1\lambda$, 0.2λ, 0.3λ and 0.4λ. Here, calculations were carried out from $\theta_i = 1°$ to $\theta_i = 90°$, since the optical theorem cannot be defined for $\theta_i = 0$. The error $|1 - \sum_n P_n|$ is less than 10^{-4} when σ_h is 0.3λ or less. When $\sigma_h = 0.4\lambda$, the error is still less than 4×10^{-3}. Therefore, we may say that our approach gives a reasonable solution if $\sigma_h \leqq 0.4\lambda$. However, further numerical studies are required for various sets of parameters W, L and σ_h to clarify the validity and limitation of the present formalism as a computational technique.

Figure 3 illustrates the scattering cross section against the scattering angle θ_s for $\sigma_h = 0.1\lambda$, 0.2λ, 0.3λ and 0.4λ, where $\theta_i = 60°$. We see major peaks at scattering angles $\theta_s \approx 45.6°$, $72.5°$, $95.7°$, $120.00°$ and $154.2°$, which agree with diffraction angles calculated by the grating formula (53) below. When $\sigma_h = 0.1\lambda$ (Fig. 3(A)), the -1st diffraction beam at $\theta_s \approx 95.7°$ is the largest among diffraction beams, but the 0-order diffraction beam becomes the largest when $\sigma_h = 0.2\lambda$, 0.3λ and 0.4λ (See Figs. 3(B)–(D)). As is expected by (39), we see in Fig. 4 that the scattering cross section vanishes when the scattering angle becomes grazing.

Figure 4 illustrates relative powers of diffraction against the angle of incidence θ_i for $\sigma_h = 0.1\lambda$, 0.2λ, 0.3λ and 0.4λ. When $\sigma_h = 0.1\lambda$ (Fig. 4(A)), the -1st diffraction is the largest for any angle of incidence $1° \leqq \theta_i \leqq 90°$. However, the 0-order diffraction increases when σ_h goes up (See Figs. 4(B)–(D)). Here, we note that Fig. 3(A) for $\sigma_h = 0.1\lambda$ agrees well with our previous results [3] based on the Rayleigh hypothesis.

5. Comparison with Periodic Case

When $W \to \infty$, the finite periodic surface becomes a periodic surface with $z = f(x) = f_p(x)$. To discuss a relation between a finite periodic case and such a periodic case, we consider the wave diffraction by such a periodic surface.

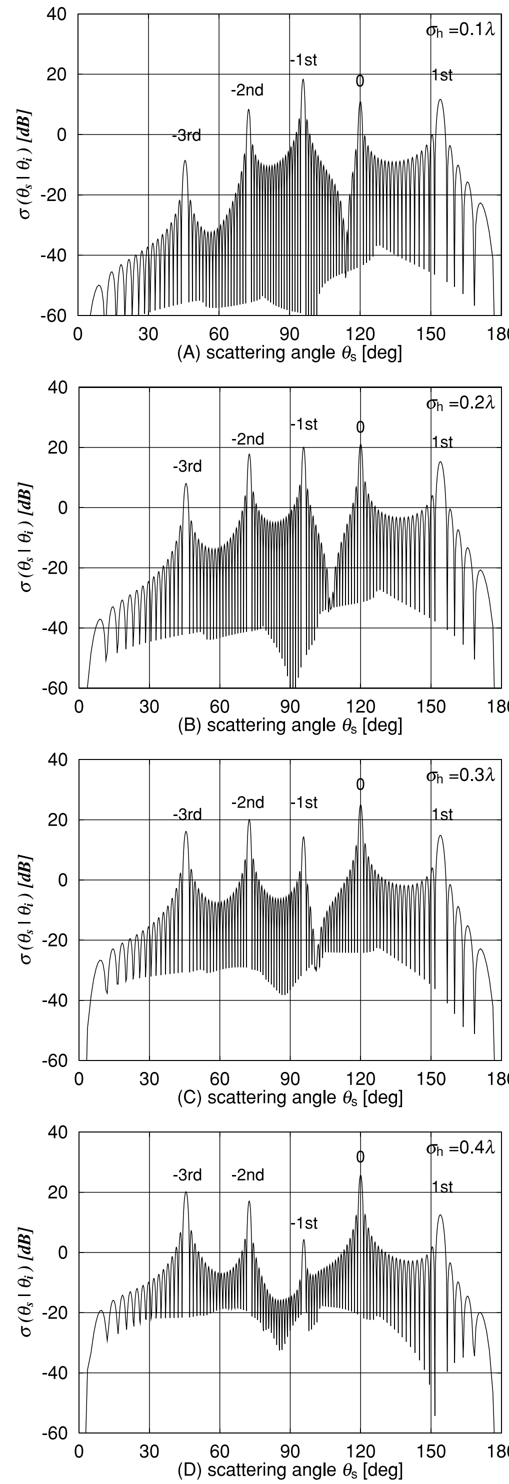

Fig. 3 Scattering cross section $\sigma(\theta_s|\theta_i)$. The angle of incidence is $\theta_i = 60$. Period $L = 2.5\lambda$, width $W = 50\lambda$. σ_h is the surface height and λ is wavelength. $N_B = 160$, $k_B/k_L = 8.5$. (A) $\sigma_h = 0.1\lambda$, (B)$\sigma_h = 0.2\lambda$, (C)$\sigma_h = 0.3\lambda$, (D)$\sigma_h = 0.4\lambda$.

Since the surface corrugation is periodic with infinite extent, the wave field has the Floquet form, which we write as

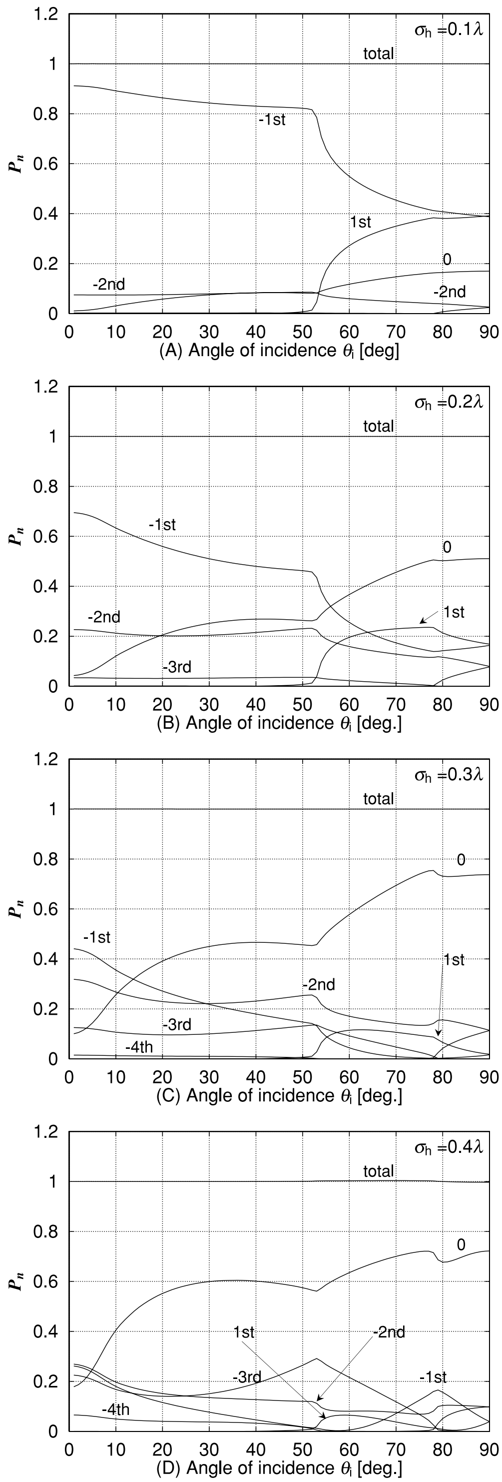

Fig. 4 Optical theorem against the angle of incidence θ_i for a finite sinusoidal surface with period $L = 2.5\lambda$ and width $W = 50\lambda$. λ is wavelength. P_n is relative power of diffraction with discrete index and total means $\sum_n P_n$. σ_h is the surface height. $N_B = 160$, $k_B/k_L = 8.5$. (A) $\sigma_h = 0.1\lambda$, (B)$\sigma_h = 0.2\lambda$, (C)$\sigma_h = 0.3\lambda$, (D)$\sigma_h = 0.4\lambda$.

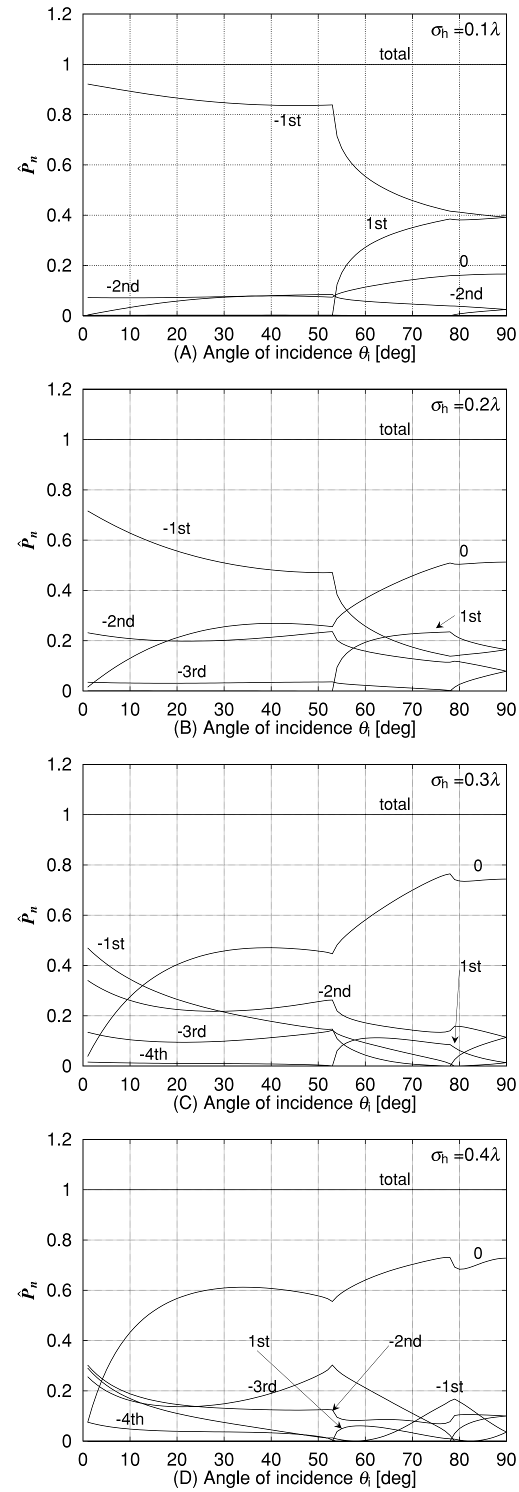

Fig. 5 Optical theorem against the angle of incidence θ_i for a sinusoidal surface with infinite extent. period $L = 2.5\lambda$. $\hat{P}_n$ is relative power of the nth order diffraction and total means $\sum_n \hat{P}_n$. σ_h is the surface height and λ is wavelength. (A) $\sigma_h = 0.1\lambda$, (B)$\sigma_h = 0.2\lambda$, (C)$\sigma_h = 0.3\lambda$, (D)$\sigma_h = 0.4\lambda$.

$$\psi(x,z) = e^{-ipx}[e^{-i\beta_0(p)z} - e^{i\beta_0(p)z}] + e^{-ipx}\sum_{n=-\infty}^{\infty}\hat{A}_n e^{-ink_Lx+i\beta_n(p)z}. \quad (52)$$

This relation holds in the region with $z > \max\{f(x)\}$. Here, $\hat{A}_n$ is the amplitude of the nth order Floquet mode, which is diffracted into the direction θ_n determined by the grating formula [7]:

$$\cos(\theta_n) = -\cos(\theta_i) - nk_L/k. \quad (53)$$

According to Ref. [3], we write the optical theorem as

$$\sum_{n=-\infty}^{\infty}\hat{P}_n = 1, \quad \hat{P}_n = \frac{Re[\beta_n(p)]|\hat{A}_n|^2}{2\beta_0(p)Re[\hat{A}_0]}, \quad (54)$$

where $\hat{P}_n$ is the relative power of the nth order diffraction.

For the sinusoidal surface (40), we determined the amplitude $\hat{A}_m$ by a non-Rayleigh method in [4], where error $|1-\sum_n \hat{P}_n|$ is less than 1×10^{-4} even when $\sigma_h = 0.4\lambda$. Then, we calculated relative powers $\hat{P}_n$, which are illustrated in Fig. 5 against the angle of incidence for several values of σ_h.

Comparing Fig. 4 with Fig. 5, one finds that relative powers P_n for the finite periodic case are much similar to $\hat{P}_n$ of the infinite periodic case. When $\sigma_h = 0, 1\lambda$ and 0.2λ, however, we see some difference at $\theta_i \approx 0°$ and $53.13°$, which are critical angles of incidence. At $\theta_i \approx 53.13°$, P_n varies gradually against θ_i but $\hat{P}_n$ changes rapidly. Such similarity and difference were demonstrated in a previous paper [3] but only for a small value of σ_h. When $\sigma_h = 0.4\lambda$, the difference at $\theta_i \approx 53.13°$ is much reduced but the difference at $\theta_i \approx 0°$ still remains. This is probably caused by physical nature of solutions: the scattered wave $\psi_s(x,z)$ satisfies the radiation condition at $x \to \pm\infty$ but the diffracted wave described by the Floquet form does not satisfy the radiation condition.

From these examples, we may say that the relative power of diffraction beam P_n and the relative power of diffraction $\hat{P}_n$ could become a bridge connecting the scattering from a finite periodic surface and the diffraction by a periodic surface, when the angle of incidence is not close to either a critical angle or a grazing angle.

6. Conclusion

Modifying the spectral formalism to the scattering from a rough surface [5], we have presented a formulation suitable for the case of a finite periodic surface. For a sinusoidally corrugated surface, we demonstrated by numerical calculations that our formulation gives a reasonable solution. We gave discussions on an expectation such that the relative powers of diffraction beam for a finite periodic case are much similar to the relative power of diffraction for a perfectly periodic case. We found good similarity in numerical examples when the angle of incidence is not close to either a critical angle or a grazing angle.

However, we note that such an expectation came from physical insight but no theoretical background exists [3]. Such an expectation is not obvious physically, because the wave scattered from a finite periodic surface satisfies the radiation condition at $x \to \pm\infty$ but the diffracted wave described by the Floquet form does not.

We are interested in seeing whether such an expectation works or not for a TM wave case [8] and for other structures such as finite array of grooves [9] or slits [10]. Also, further numerical studies are required to clarify the limitation of the present formalism as a computational technique. These problems, however, are left for future study.

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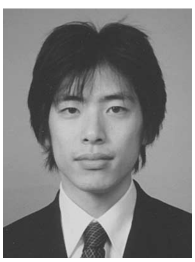

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