PAPER

Wave Scattering and Diffraction from a Finite Periodic Surface: Diffraction Order and Diffraction Beam

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SUMMARY This paper deals with a mathematical formulation of the scattering from a periodic surface with finite extent. In a previous paper the scattered wave was shown to be represented by an extended Floquet form by use of the periodic nature of the surface. This paper gives a new interpretation of the extended Floquet form, which is understood as a sum of diffraction beams with diffraction orders. Then, the power flow of each diffraction beam and the relative power of diffraction are introduced. Next, on the basis of a physical assumption such that the wave scattering takes place only from the corrugated part of the surface, the amplitude functions are represented by the sampling theorem with unknown sample sequence. From the Dirichlet boundary condition, an equation for the sample sequence is derived and solved numerically to calculate the scattering cross section and optical theorem. Discussions are given on a hypothesis such that the relative power of diffracted beam becomes almost independent of the width of surface corrugation.

key words: *wave scattering, finite periodic surface, diffraction beam, sampling theorem*

1. Introduction

The scattering and diffraction by periodic gratings have received much interest, because they have important applications such as optical filters, leaky wave antennas, and waveguide couplers. Many works have been carried out to investigate diffraction and scattering characteristics of a grating with infinite extent [1], [2], a semi-infinite grating [3], [4] and a finite grating [5]–[9]. In case of an infinite grating, the incident plane wave is scattered into only discrete directions and the scattered wave becomes a sum of diffracted plane waves with discrete index known as the diffraction order. The power carried by each diffracted wave is commonly investigated to calculate the diffraction efficiency. On the other hand, in case of a finite grating the scattering into all directions takes place and hence discussions have been concentrated on the angular distribution of the scattering. But no mathematical concept has been presented for the diffraction order or diffracted wave with discrete index, though a finite grating is used as a device diffracting the incident wave into discrete directions in practical applications. This implies that a periodic case with infinite extent and a finite periodic case have been considered to be entirely different in mathematical formulation.

To bridge gaps between an infinite extended and a finite periodic cases, however, we introduced the periodic Fourier transform as a new idea of analysis [10], [11]. We then found out that the scattered wave has an extended Floquet form in a previous paper. This paper gives a new interpretation of the extended Floquet form, which is understood as a sum of orthogonal diffraction beams with diffraction orders. Then, the power flow of a diffraction beam, the optical theorem and the relative power of diffraction are defined. We present a new hypothesis such that the relative power of diffracted beam becomes almost independent of the width of surface corrugation. The validity of this hypothesis is discussed numerically.

Next, taking a physical assumption such that the wave scattering takes place only from the corrugated part of the surface, this paper derives another representation of the scattered wave by the sampling theorem, where the amplitude functions are represented by a sample sequence. In case of TE wave incident on a sinusoidal surface with finite extent, the sample sequence is determined numerically to calculate the scattering cross section and optical theorem.

2. Diffraction Beams

Let us consider the wave scattering from a periodic surface with finite extent (See Fig. 1). We write the finite periodic deformation as

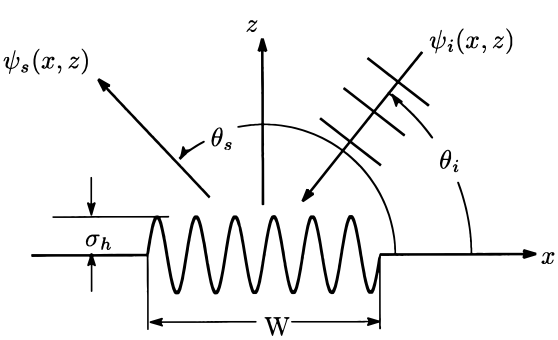

Fig. 1 Scattering and diffraction by a finite periodic surface. $\psi_i(x,z)$ and $\psi_s(x,z)$ denote the incident plane wave and scattered wave, respectively. θ_i is the angle of incidence and θ_s is a scattering angle. W and σ_h are the width and height of periodic corrugation, respectively.

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$$z = f(x) = u(x|W) f_p(x), \tag{1}$$
$$f_p(x) = f_p(x+L), \tag{2}$$

where $f_p(x)$ is a periodic function with a period L and $u(x|W)$ is a rectangular pulse given by

$$u(x|W) = u^2(x|W) = \begin{cases} 1, & |x| \le W/2 \\ 0, & |x| > W/2 \end{cases}, \tag{3}$$

Here, W is the width of the periodic corrugation. By k_W we denote the spatial angular frequency associated with W and by k_L the spatial angular frequency associated with L,

$$k_W = \frac{2\pi}{W}, \quad k_L = \frac{2\pi}{L}, \quad N_{WL} = \frac{W}{L}, \tag{4}$$

where N_{WL} is the ratio of W to the period L and is implicitly assumed to be an integer. Note that (1) becomes periodic with infinite extent when $W \to \infty$.

We write the y-component of the electric filed by $\psi(x,z)$, which satisfies the wave equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right]\psi(x,z) = 0, \tag{5}$$

in the region with $z > f(x)$ and the Dirichlet condition

$$\psi(x,z) = 0, \quad z = f(x), \tag{6}$$

on the surface (1). Here, $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength. We write the incident plane wave $\psi_i(x,z)$ as

$$\psi_i(x,z) = e^{-ipx}e^{-i\beta_0(p)z}, \quad p = k \cdot \cos\theta_i, \tag{7}$$

$$\beta_m(p) = \sqrt{k^2 - (p+mk_L)^2}, \quad \mathrm{Im}\left[\beta_m(p)\right] \ge 0, \quad (m = 0, \pm 1, \pm 2, \cdots), \tag{8}$$

where θ_i is the angle of incidence and Im stands for imaginary part.

We write $\psi(x,z)$ as

$$\psi(x,z) = e^{-ipx}\left[e^{-i\beta_0(p)z} - e^{i\beta_0(p)z}\right] + \psi_s(x,z), \tag{9}$$

which is a sum of three components: the incident plane wave, the specularly reflected wave and $\psi_s(x,z)$ the scattered wave due to the surface corrugation. One may determine a form of the scattered wave by use of the periodic nature of the surface corrugation [10]. Then, the scattered wave is shown to have the extended Floquet form:

$$\psi_s(x,z) = \frac{e^{-ipx}}{k_L}\sum_{m=-\infty}^{\infty} e^{-imk_Lx} \times \int_{-\pi/L}^{\pi/L} A_m(s) e^{-isx+i\beta_m(p+s)z} ds \tag{10}$$

$$= \sum_{m=-\infty}^{\infty} \Psi_m(x,z), \tag{11}$$

which holds for $z > \max\{f(x)\}$. Here, $\Psi_m(x,z)$ is the m-th order diffraction beam

$$\Psi_m(x,z) = \frac{1}{k_L}\int_{-\pi/L}^{\pi/L} A_m(s) \times e^{-i(p+s+mk_L)x + i\beta_m(p+s)z} ds. \tag{12}$$

We note that (10) satisfies (5) and the radiation condition at $z \to \infty$ term by term. $A_m(s)$ is a complex amplitude of a plane wave propagating into a direction $\mathbf{k} = -(s+p+mk_L)\mathbf{e}_x + \beta_m(p+s)\mathbf{e}_z$, $\mathbf{e}_x$ and $\mathbf{e}_z$ being unit vectors into the x and z directions, respectively. In other word, $A_m(s)$ is the amplitude of a plane wave scattered into the θ_s direction, where θ_s is given by

$$\cos\theta_s = -\left(\cos\theta_i + \frac{s}{k} + m\frac{\lambda}{L}\right). \tag{13}$$

If we put $s = 0$, this relation becomes the famous grating formula:

$$\cos\theta_m = -\left(\cos\theta_i + m\frac{\lambda}{L}\right), \tag{14}$$

where m stands for a diffraction order.

Let us discuss on the representation (10). The representation (10) is a product of the exponential phase factor e^{-ipx} and a harmonic series that is a 'Fourier series' with 'Fourier coefficients' given by band-limited Fourier integrals. When W goes to infinity and (1) becomes periodic with infinite extent, the amplitude function becomes [10]

$$A_m(s) = k_L \hat{A}_m \delta(s), \quad (W \to \infty), \tag{15}$$

where $\hat{A}_m$ is a diffraction amplitude and $\delta(s)$ is Dirac's delta function. Hence, (10) is reduced to the famous Floquet form for the periodic grating (See Eq. (39) below). Therefore, (10) is regarded as an extension of the Floquet form and is called the extended Floquet form [10], [11]. When W is much larger than the wavelength, however, $A_m(s)$ is expected to be a localized function taking a large value only at $s \approx 0$. This implies that $\Psi_m(x,z)$ is a beam, of which main lobe is scattered into the direction θ_m determined by (14). Thus, we have call $\Psi_m(x,z)$ the m-th order diffraction beam. The concept of such a diffraction beam with diffraction order for a finite periodic case is first proposed in this paper. By a direct calculation, one easily finds an orthogonality relation between such diffraction beams,

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi_m(x,z)\Psi_n^*(x,z)dx = \delta_{m,n}\int_{-k_L/2}^{k_L/2}\left|\frac{A_m(s)}{k_L}\right|^2 e^{-2\mathrm{Im}[\beta_m(p+s)]z} ds, \tag{16}$$

where $z > \max\{f(x)\}$ is implicitly assumed and $\delta_{m,n}$

is Kroneckers's delta. From this relation, we understand (11) as a sum of orthogonal diffraction beams with diffraction orders.

We have defined the diffraction beam with any order. Next, we calculate the power flow carried by a diffraction beam. From (12), one easily obtain a relation of the power flow into the z direction,

$$\mathrm{Re}\int_{-\infty}^{\infty}\frac{\partial\Psi_m(x,z)}{i\partial z}\Psi_n^*(x,z)dx=\delta_{m,n}\Phi_m,$$
$$\Phi_m=2\pi\int_{-k_L/2}^{k_L/2}\mathrm{Re}\left[\frac{\beta_m(p+s)}{k_L^2}\right]|A_m(s)|^2ds, \tag{17}$$

where Φ_m is the diffraction power carried by the m-th order diffraction beam. The asterisk and Re stand for complex conjugate and real part, respectively.

3. Optical Theorem

Since the plane wave is incident on a surface with infinite extent, the total power of incidence becomes infinite. However, the scattered power always remains finite, because the periodic surface is finite in extent. Concerning such a finite power of scattering, we may obtain an optical theorem [10],

$$\begin{aligned}\frac{4\pi}{k_L}\beta_0(p)\mathrm{Re}[A_0(0)] &= \frac{kW}{2\pi}\int_0^{\pi}\sigma(\theta_s|\theta_i)d\theta_s,\\ &=\sum_{n=-\infty}^{\infty}\Phi_m,\end{aligned} \tag{18}$$

where $4\pi\beta_0(p)\mathrm{Re}[A_0(0)]/k_L$ is the total power of scattering, Φ_m is the power carried by the m-th order diffraction beam (17) and $\sigma(\theta_s|\theta_i)$ is the differential scattering cross section divided by W,

$$\begin{aligned}\sigma(\theta_s|\theta_i)=\frac{(2\pi)^2k}{k_L^2W}\sum_{m=-\infty}^{\infty}\sin^2\theta_s u(-k\cos\theta_s-p\\ -mk_L|k_L)|A_m(-k\cos\theta_s-p-mk_L)|^2.\end{aligned} \tag{19}$$

The relation (18) means that the scattering takes place with the loss of specular reflection.

Assuming $\beta_0(p)=k\sin(\theta_i)\neq 0$, we rewrite the optical theorem as

$$\sum_{m=-\infty}^{\infty}P_m=1,\quad P_m=\frac{k_L\cdot\Phi_m}{4\pi\beta_0(p)\mathrm{Re}[A_0(0)]}, \tag{20}$$

where P_m is the relative power of the m-th order diffraction beam divided by the total scattering power $4\pi\beta_0(p)\mathrm{Re}[A_0(0)]/k_L$. Roughly speaking, the total scattering power and the diffraction power Φ_m are proportional to W, because the scattering takes place from the corrugation with width W. This implies a hypothesis such that the relative power P_m becomes almost independent of W. We will see the validity of this hypothesis later.

4. Expression by Sampling Theorem

By use of the periodic nature of the surface, we have derived the extended Floquet form. However, taking a fact that the scattered wave vanishes at the flat part of the surface (1), this section derives another representation of the scattered wave by the sampling theorem, where the amplitude functions are represented by a sample sequence.

We assume the Rayleigh hypothesis and the extended Floquet from (11) are valid even on the corrugated part of the surface (1). When the corrugation height is sufficiently small, this assumption may give reasonable results for the scattering cross section.

Substituting (9), (11), (12) into the boundary condition (6), we then obtain

$$\begin{aligned}\frac{1}{k_L}\sum_{m=-\infty}^{\infty}e^{-imk_Lx}\int_{-\pi/L}^{\pi/L}A_m(s)e^{-isx}ds\\ =u(x|W)Q(x),\end{aligned} \tag{21}$$

$$\begin{aligned}Q(x)=2i\sin[\beta_0(p)f_p(x)]-\frac{1}{k_L}\sum_{m=-\infty}^{\infty}e^{-imk_Lx}\\ \times\int_{-\pi/L}^{\pi/L}A_m(s)e^{-isx}\left[e^{i\beta_m(p+s)f_p(x)}-1\right]ds.\end{aligned} \tag{22}$$

Here, the left hand side of (21) is equal to $e^{ipx}\psi_s(x,0)$. Due to the boundary condition (6), the right-hand side of (21) vanishes for $|x|>W/2$. For $|x|\le W/2$, we represent $Q(x)$ by Fourier series as

$$Q(x)=\sum_{n=-\infty}^{\infty}Q_n\cdot e^{-inK_Wx},\quad |x|\le W/2, \tag{23}$$

$$Q_n=\frac{1}{W}\int_{-W/2}^{W/2}Q(x)e^{inK_Wx}dx. \tag{24}$$

If we calculate the Fourier transform of (21) and we take the relation (23), we obtain a representation of $A_m(s)$ by the sampling theorem,

$$A_m(s)=\frac{1}{L}\sum_{n=-\infty}^{\infty}Q_nU(s-nk_W+mk_L|W), \tag{25}$$

where $U(s|W)$ is the Fourier transform of $u(x|W)$,

$$U(s|W)=\int_{-\infty}^{\infty}u(x|W)e^{isx}dx=\frac{\sin(Ws/2)}{(s/2)}, \tag{26}$$

$$U(l\cdot k_W|W)=W\delta_{l,0},\quad (l=0,\pm1,\pm2,\cdots), \tag{27}$$
$$\lim_{W\to\infty}U(s|W)=2\pi\delta(s). \tag{28}$$

In (25) Q_n is related with a sampled value of $A_m(s)$. Putting $s=nk_W-mk_L$ in (25), one easily finds a relation

$$A_m(nk_W - mk_L) = \frac{W}{L} Q_n. \quad (29)$$

This means that Q_n is an amplitude factor of another beam scattered into the direction θ_n:

$$\cos\theta_n = -\left(\cos\theta_i + \frac{\lambda}{L}\frac{n}{N_{WL}}\right). \quad (30)$$

By the relation (25), the scattering problem is reduced to finding a sampled sequence $\{Q_n\}$. Let us obtain an equation for Q_n. Substituting (25) into (22) and (24), we obtain a linear equation for Q_n,

$$Q_n + \sum_{l=-\infty}^{\infty} D_{nl} Q_l = C[nk_W|\beta_0(p)] - C[nk_W| - \beta_0(p)], \quad (31)$$

where D_{nl} and $C[s|\beta]$ are

$$D_{nl} = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\int_{-k_L/2}^{k_L/2} U(s + mk_L - lk_W|W) \times C[nk_W - mk_L - s|\beta_m(p+s)]ds, \quad (32)$$

$$C[s|\beta] = \frac{1}{W}\int_{-W/2}^{W/2} e^{isx}\left[e^{i\beta f_p(x)} - 1\right]dx. \quad (33)$$

In what follows, (31) is solved numerically to obtain the sequence $\{Q_n\}$.

However, the diffraction becomes strong into the directions given by (14) and $A_m(s)$ takes a large value only at $s \approx 0$. Therefore, Q_n is expected to be large only when n is an integer multiple of N_{WL}. This fact suggests an approximation such that Q_n is neglected when n is not equal to an integer multiple of N_{WL},

$$Q_n \approx 0, \quad (n \neq lN_{WL}, \quad l = 0, \pm 1, \pm 2, \cdots). \quad (34)$$

If this approximation (34) is employed, the size of matrix equation (31) may be much reduced. This approximation works well in some case. But we omit discussions on this here.

5. Numerical Example

As a simple example, we consider a case where the surface deformation is sinusoidal

$$f_p(x) = \sigma_h \cdot \sin(k_L x), \quad (35)$$

where σ_h is the height of corrugation and is assumed much smaller than λ the wavelength. To make $f(x)$ a continuous function of x at $x = \pm W/2$, N_{WL} has to be a positive integer or a half-integer. However, we simply assume N_{WL} is an integer. In case of the sinusoidal corrugation, (33) becomes

$$C[s|\beta] = \frac{1}{W}\sum_{l=-\infty}^{\infty}\left[J_l(\sigma_h\beta) - \delta_{l,0}\right]U(s + lk_L|W), \quad (36)$$

where we have used the formula on Bessel function $J_l(\cdot)$,

$$e^{i\sigma_h\beta\sin(k_L x)} = \sum_{l=-\infty}^{\infty} J_l(\sigma_h\beta)\, e^{ilk_L x}. \quad (37)$$

For numerical calculation, we put

$$\sigma_h = 0.1\lambda, \quad L = 2.5\lambda, \quad W = 20L = 50\lambda. \quad (38)$$

We take truncations such that the sequence $\{Q_n\}$ and the matrix $[D_{nl}]$ in (31) are approximated by a 201-dimensional vector and by a 201×201 matrix, respectively. We also replace the infinite sum in (32) by a finite sum from $m = -6$ to 6 to calculate an element of $[D_{nl}]$.

Then, we numerically solved (31) to get Q_n shown in Fig. 2. We see in Fig. 2 that $|Q_n|$ becomes large only when n is an integer multiple of $N_{WL} = 20$, as is expected. We substitute Q_n into (25) to obtain the complex amplitude $A_m(s)$. Using (19), we then obtain the differential cross section $\sigma(\theta_s|\theta_i)$ against θ_s illustrated in Fig. 3, which agrees well with results in

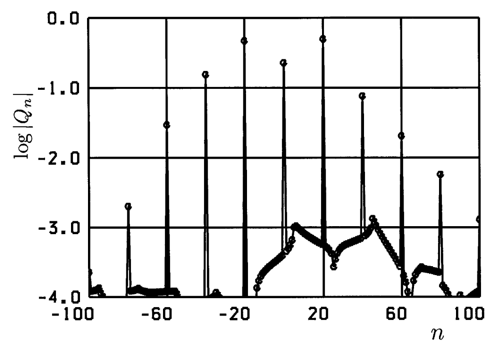

Fig. 2 $\log_{10}|Q_n|$ against n. $\theta_i = 60°$, $L = 2.5\lambda$, $W = 20L$, $\sigma_h = 0.1\lambda$ and $N_{WL} = 20$. $|Q_n|$ becomes large when n is an integer multiple of $N_{WL} = 20$.

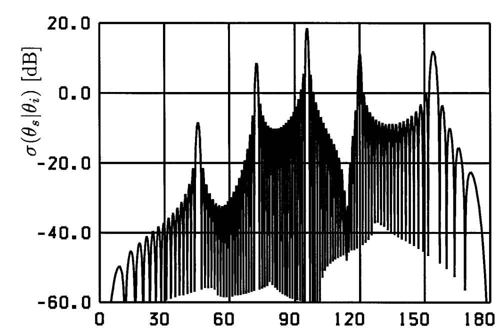

Fig. 3 Differential scattering cross section $\sigma(\theta_s|\theta_i)$. θ_s and θ_i are a scattering angle and the angle of incidence, respectively. $\theta_i = 60°$, $L = 2.5\lambda$, $W = 20L$, $\sigma_h = 0.1\lambda$ and $N_{WL} = 20$.

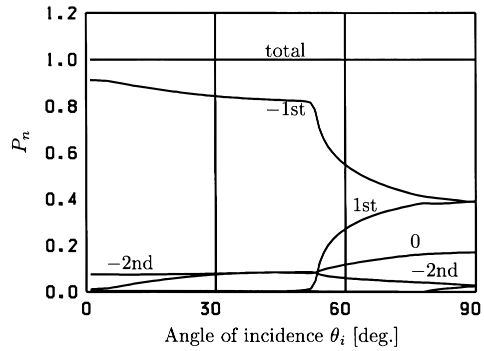

Fig. 4 Relative power P_n for periodic surface with finite extent. $L = 2.5\lambda$, $W = 20L$, $\sigma_h = 0.1\lambda$. The 1st and 2nd order continuous Floquet modes become propagating about $\theta_i \approx 53.13°$ and 78.46°, respectively.

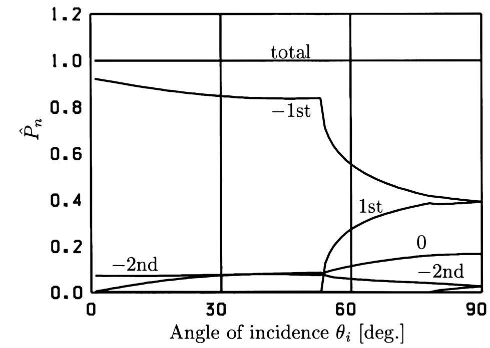

Fig. 5 Relative power $\hat{P}_n$ for periodic surface with infinite extent. $L = 2.5\lambda$, $\sigma_h = 0.1\lambda$. The 1st and 2nd order Floquet modes become propagating at $\theta_i \approx 53.13°$ and 78.46°, respectively.

Ref. [11]. Due to the surface periodicity, the scattering cross section has major peaks at $\theta_s = 45.6°$, 72.5°, 95.7°, 120.0°, and 154.2°. These angles coincide with the diffraction angles calculated by the grating formula (14). In Fig. 3 the −1st and 1st order diffraction are relatively large, because the surface deformation is sinusoidal. Sidelobes around major peaks are physically caused by interference between waves scattered by the edges at $x = \pm W/2$. However, the sidelobe levels are much reduced by apodisation [12].

From Q_n, we calculate the optical theorem (20), which is illustrated as a function of the angle of incidence in Fig. 4. When $\sigma_h = 0.1\lambda$, total power $\sum P_n$ is almost equal to 1. In fact, the error $|1 - \sum P_n|$ is less than 5×10^{-5} for $\theta_i \geq 1°$ in case of (38). The error is less than 1×10^{-4} even when $\sigma_h = 0.2\lambda$. However, we note that, when $\sigma_h > 0.2\lambda$, the error becomes much large and Rayleigh hypothesis may not be useful for calculating the scattering cross section.

6. Comparison with Periodic Case

We have present a hypothesis such that the relative power P_m becomes almost independent of W. To see the validity of this hypothesis, we compare a finite periodic case to a periodic case with $W = \infty$.

When $W \to \infty$, (1) becomes a sinusoidal surface with infinite extent. We denote the wave field for such a periodic case by $\psi_D(x, z)$, which is represented by the Floquet form. From (10) and (15), however, we write the Floquet form as

$$\begin{aligned}\psi_D(x,z) &= e^{-ipx}e^{-i\beta_0(p)z} - e^{-ipx}e^{i\beta_0(p)z} \\ &+ e^{-ipx}\sum_{m=-\infty}^{\infty}\hat{A_m}e^{-imk_Lx}e^{i\beta_m(p)z}, \quad (39)\end{aligned}$$

which is a sum of three components: the incident plane wave, the specularly reflected wave and the diffracted waves with amplitudes $\{\hat{A}_m\}$. This definition of diffracted wave is different from others; for example, Refs. [1], [2] consider the wave field as a sum of two components: the incident plane wave and the diffracted wave, where $\hat{A}_m - \delta_{m0}$ in (39) is regarded as the amplitude of the m-th order Floquet mode. Notice that $\hat{A}_m$ is defined as the amplitude of the m-th order Floquet mode in this paper.

In case of a periodic grating with infinite extent, the energy conservation law may be written as

$$\beta_0(p) = \sum_{m=-\infty}^{\infty} \mathrm{Re}\left[\beta_m(p)\right]\left|\hat{A_m} - \delta_{m,0}\right|^2. \quad (40)$$

Rewriting this, we obtain the optical theorem

$$\sum_{m=-\infty}^{\infty}\hat{P_m} = 1, \quad \hat{P_m} = \frac{\mathrm{Re}[\beta_m(p)]|\hat{A_m}|^2}{2\beta_0(p)\mathrm{Re}[\hat{A}_0]},$$

where $2\beta_0(p)\mathrm{Re}[\hat{A}_0]$ is the total diffraction and $\hat{P_m}$ is the relative power of the m-th order Floquet mode.

By use of the non-Rayleigh method in Ref. [2], we numerically calculate the amplitude $\hat{A}_m$, from which the optical theorem in Fig. 5 is obtained. Comparing Fig. 5 with Fig. 4 for finite periodic case, one finds that the relative powers against the angle of incidence are much similar in these two cases. Let us see some numerical examples of the relative powers. When $\theta_i = 60°$, we have $P_{-3} = 1.5724228 \times 10^{-3}, (\hat{P}_{-3} = 1.5685676 \times 10^{-3})$, $P_{-2} = 5.9636250 \times 10^{-2}, (\hat{P}_{-2} = 5.7857707 \times 10^{-2})$, $P_{-1} = 0.5490089, (\hat{P}_{-1} = 0.5544853)$, $P_0 = 0.1176858, (\hat{P}_0 = 0.1138783)$, and $P_1 = 0.2720708, (\hat{P}_1 = 0.2722101)$. In case of $\theta_i = 30°$, we have $P_{-5} = 5.4440346 \times 10^{-7}, (\hat{P}_{-5} = 0.0)$, $P_{-4} = 4.8057391 \times 10^{-5}, (\hat{P}_{-4} = 2.1914748 \times 10^{-5})$, $P_{-3} = 2.7040027 \times 10^{-3}, (\hat{P}_{-3} = 2.6856598 \times 10^{-3})$, $P_{-2} = 7.9067543 \times 10^{-2}, (\hat{P}_{-2} = 7.6232642 \times 10^{-2})$, $P_{-1} = 0.8429495, (\hat{P}_{-1} = 0.8474179)$ and $P_0 = 7.5199872 \times 10^{-2}, (\hat{P}_0 = 7.3641948 \times 10^{-2})$. These examples show the hypothesis is reasonable. But there is some difference in these figures such that the relative powers rapidly change at $\theta_i \approx 53.13°$ in the periodic case of Fig. 5, but the changes are gradual in the finite periodic case in Fig. 4. Here, $\theta_i \approx 53.13°$ is a critical angle at which the 1-st order Floquet mode becomes propagating. Also, one may see slight difference at $\theta_i \approx 0$, which is a critical angle for the 0-th order Floquet mode. Such difference comes from a fact that a finite periodic surface has continuous spectrum but a periodic surface has discrete spectrum. Since the continuous spectrum approaches to the discrete one when W becomes large, we may conclude the hypothesis may be useful when W is much larger than the period L and when the angle of incidence is not critical.

7. Conclusions

We have dealt with the wave scattering from a periodic surface with finite extent. By a new interpretation of the extended Floquet form, the scattered wave is understood as a sum of orthogonal diffraction beams with diffraction orders. The power flow of a diffraction beam and the relative power of diffraction are mathematically defined as new concepts. We present a new hypothesis such that the relative power of diffracted beam becomes almost independent of the width of surface corrugation. The validity of this hypothesis is discussed briefly.

We also derived another representation of the scattered wave by the sampling theorem, where the amplitude functions are represented by a sample sequence. In case of TE wave incident on a sinusoidal surface with finite extent, the sample sequence is first truncated and then is solved numerically to calculate the scattering cross section and optical theorem. However, our discussion was limited to a TE wave case with Rayleigh hypothesis for a small corrugation height. We note that our formulation can be immediately applied to calculating the scattering cross section for a dielectric interface with a finite periodic corrugation. However, extensions to non-Rayleigh method and TM wave case are left for future study.

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