LETTER

Wave Scattering from a Periodic Surface with Finite Extent: A Periodic Approach

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SUMMARY As a method of analyzing the wave scattering from a finite periodic surface, this paper introduces a periodic approach. The approach first considers the wave diffraction by a periodic surface that is a superposition of surface profiles generated by displacing the finite periodic surface by every integer multiple of the period Λ. It is pointed out that the Floquet solution for such a periodic case becomes an integral representation of the scattered field from the finite periodic surface when the period Λ goes to infinity. A mathematical relation estimating the scattering amplitude for the finite periodic surface from the diffraction amplitude for the periodic surface is proposed. From some numerical examples, it is concluded that the scattering cross section for the finite periodic surface can be well estimated from the diffraction amplitude for a sufficiently large Λ.

key words: *wave scattering and diffraction, finite periodic surface, periodic grating*

1. Introduction

The wave scattering from a finite periodic surface has received much interest, because any real periodic structure is finite in extent. Several methods for analysis have been introduced by many authors [1]–[6]. However, we introduce here a periodic approach as an idea of analysis.

In the periodic approach, we consider a periodic surface that is a superposition of surface profiles generated by displacing the finite periodic surface by every integer multiple of the period Λ. When the period Λ goes to infinity, such a periodic surface becomes the finite periodic surface and hence the diffracted wave from such a periodic surface is physically expected to become the scattered wave from the finite periodic surface. Therefore, the scattered wave from the finite periodic surface may be well estimated from the diffracted wave for a sufficiently large Λ.

To carry out this idea of the periodic approach, we first present a modified Floquet form of the diffracted wave. We then point out a fact that the modified Floquet form becomes an integral representation of the scattered wave with a scattering amplitude function when the period Λ goes to infinity. We propose a relation estimating the scattering amplitude from diffraction amplitude in case of a large period Λ.

For several values of the period Λ, we numerically calculate the diffraction amplitude, from which the scattering amplitude and the scattering cross section are estimated. Then we find that the estimated cross section is almost independent of Λ when Λ is large enough. It is then concluded that the periodic approach is a feasible method of analysis.

2. Diffraction by Periodic Surface

Let us start with a sinusoidal surface with finite extent:

$$z = f(x) = \begin{cases} \sigma_h \cdot \sin(k_L x), & |x| \le W/2 \\ 0, & |x| > W/2 \end{cases}, \quad (1)$$

$$k_L = \frac{2\pi}{L}, \quad (2)$$

where σ_h is the corrugation height, W is the width of periodic corrugation and L is the period. Translating $f(x)$ by every integer multiple of Λ and superimposing the translated ones, we obtain the periodic surface as

$$z = f_p(x) = \sum_{n=-\infty}^{\infty} f(x + n\Lambda), \quad (3)$$

where $\Lambda \gg W$ is implicitly assumed.

We write the y component of the electric field by $\psi(x, z)$, which satisfies the wave equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right] \psi(x, z) = 0, \quad (4)$$

in free space and the boundary condition

$$\psi(x, z) = 0, \quad z = f_p(x), \quad (5)$$

on the periodic surface (3). Here, $k = 2\pi/\lambda$ is the wave number and λ is the wavelength.

We write the electric field as

$$\psi(x, z) = e^{-ipx} e^{-i\beta_0 z} - e^{-ipx} e^{+i\beta_0 z} + \psi_s(x, z), \quad (6)$$

where the first term in the right hand side is the incident plane wave, the second is the specularly reflected wave, and $\psi_s(x, z)$ is the diffracted wave due to the periodic surface corrugation (3). Here, p is related with θ_i the

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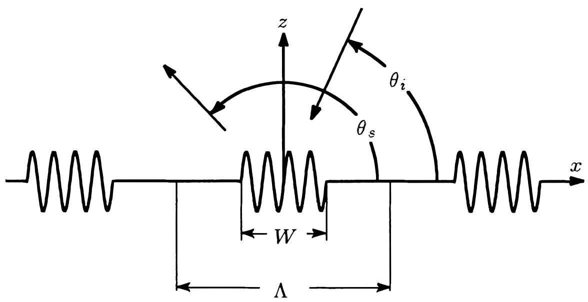


Fig. 1 Scattering and diffraction of a plane wave from a periodic surface with the period Λ. θ_i is the angle of incidence and θ_s is a scattering angle. W is the corrugation width of a partially periodic surface. When the period Λ goes to infinity, the periodic surface becomes a finite periodic surface.

angle of incidence as [See Fig. 1]

$$p = k \cdot \cos\theta_i, \tag{7}$$

and β_m is defined by

$$\beta_m = \beta(p + mk_\Lambda), \quad m = 0, \pm1, \pm2, \cdots \tag{8}$$

$$\beta(p) = \sqrt{k^2 - p^2}, \quad \mathrm{Im}\,[\beta(p)] \geq 0, \tag{9}$$

where $k_\Lambda = 2\pi/\Lambda$ and Im stands for the imaginary part.

Since the surface is periodic with the period Λ, the diffracted wave $\psi_s(x, z)$ may have the Floquet form [7], [8]:

$$\psi_s(x, z) = e^{-ipx} \sum_{m=-\infty}^{\infty} A_m e^{-imk_\Lambda x + i\beta_m z}, \tag{10}$$

which holds for $z > \sigma_h$. Here, A_m is the diffraction amplitude of the m-th Floquet mode propagating into the θ_m direction,

$$\cos(\theta_m) = -\cos(\theta_i) - \frac{mk_\Lambda}{k}, \quad k_\Lambda = \frac{2\pi}{\Lambda}. \tag{11}$$

In our periodic surface shown in Fig. 1, each periodic corrugation works as a scatterer. Since the number of scatterers per unit length is $1/\Lambda$, the diffraction amplitude A_m is expected to be proportional to $1/\Lambda$, when Λ becomes large and the wave interaction between scatteres becomes weak. Thus, it is reasonable to introduce an amplitude function $a(s)$ by the sequence $\{\Lambda A_m\}$ as

$$a(mk_\Lambda) = \Lambda A_m, \quad m = 0, \pm1, \pm2, \cdots. \tag{12}$$

The amplitude function $a(s)$ for any s will be defined by interpolation below. Even though A_m depends on Λ, $a(s)$ is almost independent of Λ for a large Λ, as will be shown below.

By use of (12), we rewrite (10) as

$$\psi_s(x, z) = \frac{e^{-ipx}}{2\pi} \sum_{m=-\infty}^{\infty} k_\Lambda \cdot a(mk_\Lambda) \times e^{-imk_\Lambda x + i\beta(p+mk_\Lambda)z}, \tag{13}$$

which we call the modified Floquet form. In (13), each term in the summation should be understood as a rectangular area given by the width k_Λ and the height $a(mk_\Lambda)\exp[-imk_\Lambda x + i\beta(p + mk_\Lambda)z]$. Thus, the summation becomes an integral when $k_\Lambda \to 0$. This fact will be used below.

Since the periodic surface and free space are lossfree, the energy conservation law holds:

$$\sum_{m=-\infty}^{\infty} \mathrm{Re}[\beta_m]|A_m - \delta_{m,0}|^2 = \beta_0, \tag{14}$$

where Re stands for the real part. The left hand side is the total diffracted power and the right hand side is the power incident on the unit surface length. By (12), however, we rewrite (14) as

$$\frac{k_\Lambda}{2\pi} \sum_{m=-\infty}^{\infty} \mathrm{Re}[\beta(p + mk_\Lambda)]|a(mk_\Lambda)|^2 = 2\mathrm{Re}[a(0)]\beta(p). \tag{15}$$

The left hand side of this also becomes an integral when $k_\Lambda = 2\pi/\Lambda \to 0$.

3. Limiting Case with $\Lambda \to \infty$

When the period Λ becomes infinitely large, the periodic surface (3) becomes a finite periodic surface (1), so that the Floquet form (10) is physically expected to become a scattered wave from such a finite periodic surface. When we take the limit $\Lambda \to \infty$, the summation in the modified Floquet form (13) may become an integral representation of the scattered wave:

$$\psi_s(x, z) = \frac{1}{2\pi} e^{-ipx} \int_{-\infty}^{\infty} a(s) e^{-isx + i\beta(p+s)z} ds. \tag{16}$$

Also, we easily find from (15) the optical theorem in case of the finite periodic surface,

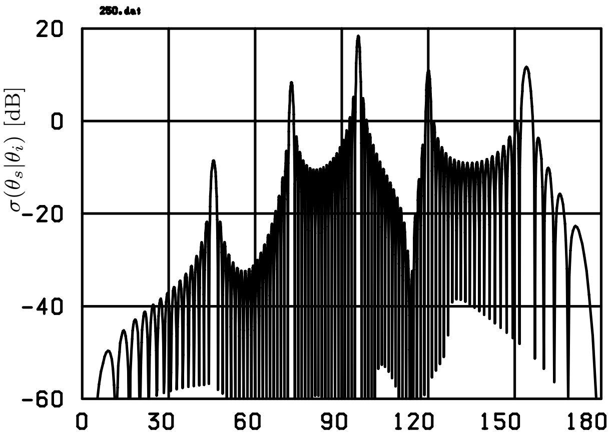
$$2\mathrm{Re}[a(0)]\beta_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} Re[\beta(p + s)]|a(s)|^2 ds \tag{17}$$

$$= \frac{kW}{2\pi} \int_0^{\pi} \sigma(\theta_s|\theta_i) d\theta_s, \tag{18}$$

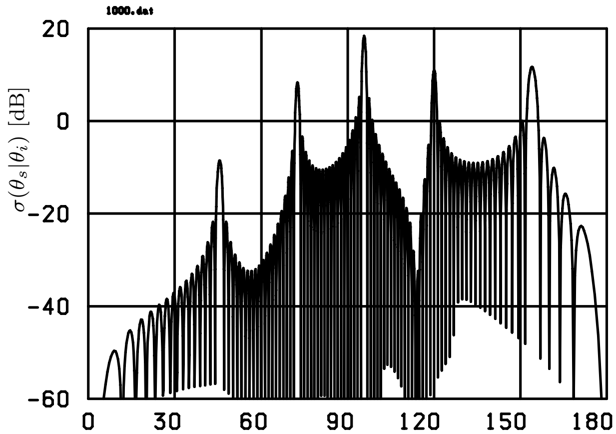
$$\sigma(\theta_s|\theta_i) = \frac{k^2 \sin^2(\theta_s)}{kW} |a(-k\cos\theta_i - k \cdot \cos\theta_s)|^2, \tag{19}$$

where $\sigma(\theta_s|\theta_i)$ is the scattering cross section per unit length [5], [6] and θ_s is a scattering angle given by [See Fig. 1]

$$k \cdot \cos(\theta_s) = -k \cdot \cos(\theta_i) - s. \tag{20}$$



(a) Scattering angle θ_s [deg]



(b) Scattering angle θ_s [deg]

Fig. 2 Scattering cross section $\sigma(\theta_s|\theta_i)$ estimated from the diffraction amplitude A_m by interpolation. $\theta_i = 60°$, $W = 20L$, $L = 2.5\lambda$, $\sigma_h = 0.1\lambda$. (a) $\Lambda = 250\lambda$, (b) $\Lambda = 1000\lambda$.

4. Estimation of the Scattering Cross Section from the Diffraction Amplitude A_m

We have shown that the modified Floquet solution becomes an integral representation of the scattered field from a finite periodic surface, when the period Λ goes to infinity. For a sufficiently large Λ, therefore, the scattering cross section may be estimated from the diffraction amplitude A_m for the periodic surface. To see the feasibility of this periodic approach, we carried out some numerical calculations.

For numerical calculations we put

$$L = 2.5\lambda, \quad W = 20L, \quad \sigma_h = 0.1\lambda, \quad \theta_i = 60°, \tag{21}$$

and $\Lambda = 250\lambda$ or $\Lambda = 1000\lambda$. By use of the method of Green's theorem in Ref. [8], we determine $[A_m]$ as a 1001-vector for $\Lambda = 250\lambda$ or a 4001-vector for $\Lambda = 1000\lambda$, with an energy error less than 0.5%. From so calculated A_m, we estimate the amplitude $a(s)$ by the interpolation formula:

$$a(s) = \lim_{M\to\infty} \sum_{m=-M}^{M} A_m \frac{\sin\left[\frac{s-mk_\Lambda}{2}\Lambda\right]}{\left[\frac{s-mk_\Lambda}{2}\right]}, \tag{22}$$

which implicitly implies $\psi(x,0) = 0$ for $|x| > \Lambda/2$. Substituting this into (19), we then calculate the scattering cross section $\sigma(\theta_s|\theta_i)$ illustrated in Fig. 2. In Fig. 2, we see peaks at scattering angles 45.6°, 72.5°, 95.7°, 120.00° and 154.2°, which are effects of diffraction by the periodic corrugation with finite extent. Figure 2(a) for $\Lambda = 250\lambda$ agrees well with Fig. 2(b) for $\Lambda = 1000\lambda$ and Fig. 2 in Ref. [5]. This fact implies that, in case of (21), the wave interaction between scatterers is negligibly small and $a(s)$ estimated from the diffraction amplitude by the interpolation (22) is almost independent of Λ, if the period Λ is larger than 250λ. Therefore, we may conclude that the periodic approach works well, if the period Λ is sufficiently large.

We have presented a basic idea of the periodic approach and some numerical results for TE wave case. The approach can be applied to the TM wave case, if the interpolation formula (22) is modified. However, a rigorous mathematical treatment and conditions under which the periodic approach works are still open question. These problems are left for future study.

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