Memoirs of the Faculty of Engineering and Design
Kyoto Institute of Technology 51 (March, 2003)

Enhanced backscattering from a disorder object: Probability distribution of backscattering intensity

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(Received September 27, 2002 ; Accepted September 27, 2002)

Abstract

A video system is developed to measure the angular distribution of light scattering from random objects. Using the system, speckle patterns were experimentally observed for disorder materials such as papers, ceramic surfaces and polystyrene surfaces. From the speckle patterns observed, the probability distribution of scattering intensities is found to become the Laplace distribution, even when the enhanced backscattering case. A belief discussion is given on the physical mechanism of the enhanced backscattering.

Key Words: *Enhanced Backscattering, Random surfaces and Random Media*

1. Introduction

When a coherent light is incident on a rigid random medium or randomly rough surface, the scattering takes place into any directions. The angular distribution of the scattering becomes a random function of scattering angle and often observed as a random image called **laser speckle.** (See Fig. 1). The laser speckle results from interference between waves scattered from different portions of the random surface or scattered from different scattereres. When the random medium or random surface is in motion, speckle pattern changes randomly in time and in space. If such changing speckle patterns are averaged in time or in ensemble sense, the averaged pattern is expected to be flat independent of scattering angle. This is theoretically true if speckle is generated by single scattering processes.

However, the averaged patterns are not flat in many cases, but have a bright spot at the direction of backscattering. An example of such phenomenon is shown in Fig. 2. This phenomenon is called the enhanced backscattering. The enhanced backscattering receives much interest, because it appears only in a disorder system, and because it may not be explained by the single scattering theory. Theoretical and numerical studies have been carried out to clarify the physical mechanism of the backscattering enhancement for a homogeneous random surface and a random medium[1][2]. Then, it is found that the enhanced backscattering is physically caused by the double path effect due to multiple

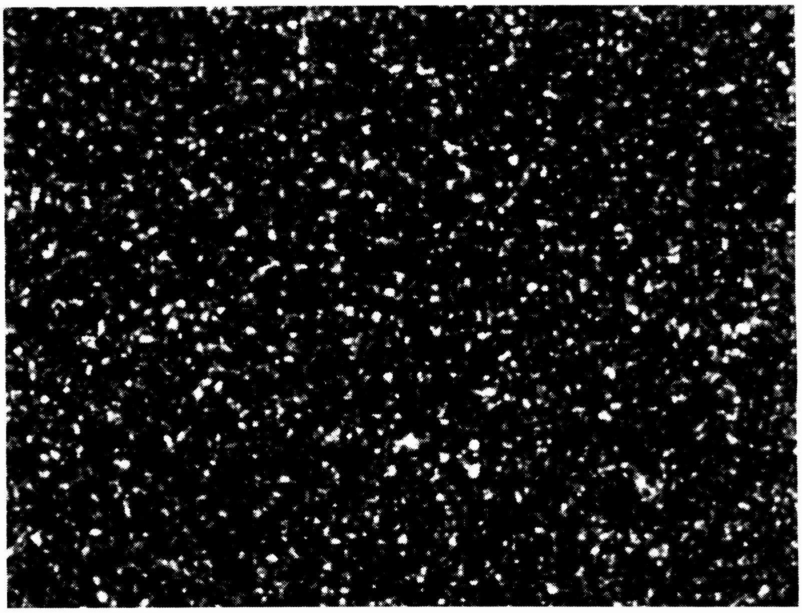

Fig. 1: Speckle pattern observed by a system in figure 4. Ceramic surface

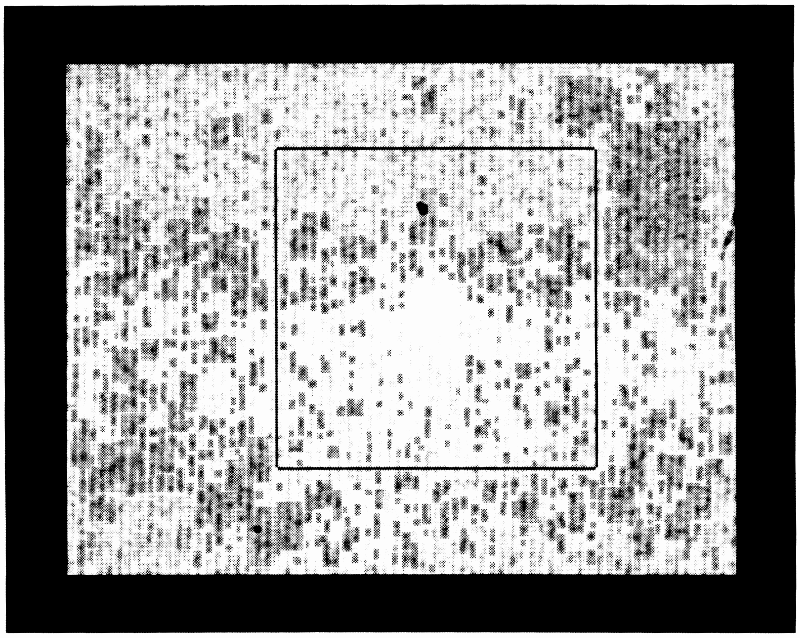

Fig. 2: Average speckle pattern. Averaged over 200 samples of spackle patterns. Ceramic surface. The box shows 256×256 pixel area, at the center of which there is a bright spot representing the enhanced backscattering. Vertical strips are caused by multiple reflection inside the half-mirror, which is a metal coated glass plate. The 256 pixel length corresponds to scattering angle width of 24.8 milli-radian.

scattering.

However, it seems that discussions so far were restricted to the average intensity of the speckle pattern[3][4]. Little work has been carried out for the probability distribution of the backscattering intensity[5]. Such the probability distribution is important in designing a measurement system, because it gives an answer to the question: how many samples of random speckle patterns are required to observe the enhanced backscattering.

In this report, we briefly discuss the physical mechanism of the enhanced backscattering in case of discrete random media made up of randomly distributed particles and a video system to measure the angular distribution of light scattering from random objects. From the speckle patterns observed, the probability distribution of scattering intensities is found to become the Laplace distribution, even in the backscattering direction.

2. Speckle and double path effect

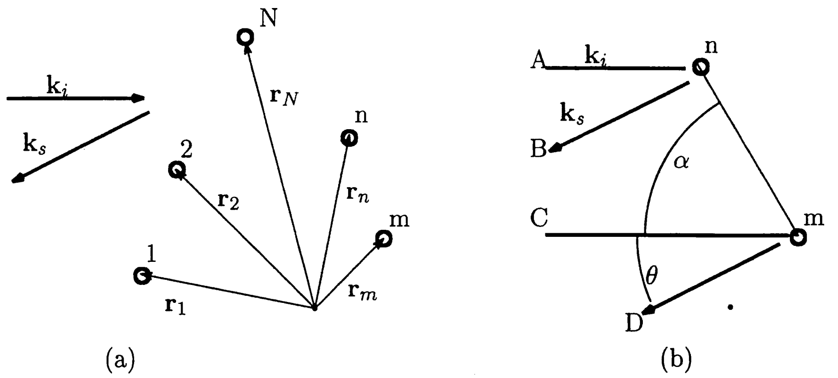

Fig. 3: (a) Scattering by randomly distributed N particles. $\mathbf{k}_0$ and $\mathbf{k}_s$ are the incident wave vector and a wave vector of scattered wave. $\mathbf{r}_i$ ($i = 1, 2, \cdots\cdots, N$) is a position vector of a particle. When the particles are randomly distributed in space, the angular distribution of the scattering becomes random. (b) If only two particles exist at $\mathbf{r}_m$ and $\mathbf{r}_n$, double scattering paths AnmD and CmnB are equal in length in the backscattering direction with $\mathbf{k}_s = -\mathbf{k}_i$ and $\theta = 0$. As a result, a constructive interference always takes place in deterministic sense at the backscattering direction.

Let us consider a simple example, where random medium is made up of particles distributed randomly. (See Fig. 3). By $\mathbf{k}_0$ and $\mathbf{k}_s$, we denote the incident wave vector and a wave vector of scattered wave. Then, the electric field $E(\mathbf{k}_s)$ scattered into $\mathbf{k}_s$ direction is given as

$$E_s(\mathbf{k}_s) \sim \sigma \sum_{n=1}^{N} E_o \exp[i(\mathbf{k}_i - \mathbf{k}_s)\cdot\mathbf{r}_n] + \sigma^2 \sum_{m,n=1,\, m\neq n}^{N} E_o \frac{e^{i(\mathbf{k}_i\mathbf{r}_m + k|\mathbf{r}_m - \mathbf{r}_n| - \mathbf{k}_s\mathbf{r}_n)}}{|\mathbf{r}_m - \mathbf{r}_n|}, \tag{1}$$

where the first term represents the single scattering effect and the second the double scattering effect. Here, $k = |\mathbf{k}_i| = |\mathbf{k}_s|$ is the wave number, N is the number of scattereres, E_o is the amplitude of the incident plane wave, and σ is scattering cross section, which is isotropic when the size of particle is much smaller than the wavelength. When the position $\mathbf{r}_n$ is randomly distributed in space, the scattered wave becomes a random function, so that the angular distribution of the scattering intensity $I_s(\mathbf{k}_s) = |E(\mathbf{k}_s)|^2$ becomes a random pattern called speckle. When $\mathbf{r}_n$ is randomly distributed and when N is large, $E_s(\mathbf{k}_s)$ is a sum of complex random variables and hence is expected to be a complex Gaussian random variable with zero average by the central limit theorem in the probability theory. Therefore, the intensity $I_s(\mathbf{k}_s) = |E(\mathbf{k}_s)|^2$ is expected to have the Laplace distribution (one side exponential distribution),

$$P(I_s(\mathbf{k}_s)) = \langle I_s(\mathbf{k}_s)\rangle \exp\left(-\frac{I_s(\mathbf{k}_s)}{\langle I_s(\mathbf{k}_s)\rangle}\right), \tag{2}$$

where $\langle I_s(\mathbf{k}_s)\rangle$ is the average intensity given by

$$\langle I_s(\mathbf{k}_s)\rangle = \langle I_s^{(1)}(\mathbf{k}_s)\rangle + \langle I_s^{(2)}(\mathbf{k}_s)\rangle. \tag{3}$$

Here, $\langle I_s^{(1)}(\mathbf{k}_s)\rangle$ and $\langle I_s^{(2)}(\mathbf{k}_s)\rangle$ are contributions from the single scattering and double scattering, respectively. From the first term in (1), we roughly estimate the single scattering effect as

$$\langle I_s^{(1)}(\mathbf{k}_s)\rangle \sim \left\langle \left| \sigma \sum_{n=1}^{N} E_o \exp[i(\mathbf{k}_i - \mathbf{k}_s)\mathbf{r}_n] \right|^2 \right\rangle = \sigma^2 E_o^2 N, \tag{4}$$

which is a constant independent of scattering angle. This means that average speckle pattern is essentially flat in case of the single scattering.

2.1 Double scattering by two scattereres

In addition to the single scattering processes, there are double scattering processes represented by the second term in (1). The double scattering processes play an important role to the enhanced backscattering. Let us consider a special case illustrated in Fig. 3 (b), where only two particles at $\mathbf{r}_m$ and $\mathbf{r}_n$ exist. We denote by $\Psi^{m,n}(\mathbf{k}_s)$ a pair of double scattering processes by the two scattereres. Then, we write $\Psi^{m,n}(\mathbf{k}_s)$ as

$$\Psi^{m,n}(\mathbf{k}_s) = \sigma^2 \left[\frac{e^{i(\mathbf{k}_i \mathbf{r}_m + k|\mathbf{r}_m - \mathbf{r}_n| - \mathbf{k}_s \mathbf{r}_n)}}{|\mathbf{r}_m - \mathbf{r}_n|} + \frac{e^{i(\mathbf{k}_i \mathbf{r}_n + k|\mathbf{r}_n - \mathbf{r}_m| - \mathbf{k}_s \mathbf{r}_m)}}{|\mathbf{r}_m - \mathbf{r}_n|} \right] E_o. \tag{5}$$

In (5), the first term $e^{i(\mathbf{k}_i \mathbf{r}_m + k|\mathbf{r}_m - \mathbf{r}_n| - \mathbf{k}_s \mathbf{r}_n)}$ is the wave, incident from C, scattered by the particle at $\mathbf{r}_m$ and scattered again by the particle at $\mathbf{r}_n$ into the direction B, which is symbolically noted by *CmnB*. On the other hand, the second term is the wave first scattered by the particle at $\mathbf{r}_n$ and scattered again by the particle at $\mathbf{r}_m$ which is denoted by *AnmD*. Let us calculate the difference l of these paths: *CmnB* and *AnmD*. From Fig. 3 (b), the difference is given by

$$l = l_{mn} \cdot [\sin(\alpha) - \sin(\alpha + \theta)] = -2 l_{mn} \cos(\alpha + \theta/2) \sin(\theta/2), \tag{6}$$

where $l_{mn} = |\mathbf{r}_m - \mathbf{r}_n|$. It should be noted that the difference l vanishes for any l_{mn} and for any α when $\theta = 0$. This means that the pair of double scattering in $\Psi^{m,n}(\mathbf{k}_s)$ is equal in phase and a constructive interference takes place in the backscattering direction with $\theta = 0$ and $\mathbf{k}_s = -\mathbf{k}_i$. This is considered as the physical reason of the enhanced backscattering, which is sometimes called the double path effect.

Assuming the angle α is distributed uniformly, we obtain the average intensity:

$$I(\theta) = \frac{1}{2\pi} \int_0^{2\pi} |\Psi_{mn}(\mathbf{k}_s)|^2 d\alpha = \frac{2\sigma^4}{l_{mn}^2} E_o^2 [1 + J_o(2k l_{mn} \sin(\theta/2))], \tag{7}$$

where J_o is the Bessel function. Since $J_o(0) = 1$, $I(\theta)$ takes the maximum value $4\sigma^4 E_o^2 / l_{mn}^2$ at the backscattering direction with $\theta = 0$. Because $J_o(1.5) = 0.51$, the half-power width θ_h of the double scattering effect is equal to $\theta_h = (3\lambda)/(2\pi \cdot l_{mn})$, which is inversely proportional to the distance l_{mn}. In other words, from the half power width θ_h, one may determine the distance l_{mn}. If l_{mn} is randomly distributed in a certain range much greater than the wavelength, $\langle I(\theta)\rangle$ averaged over l_{mn} may become the first term in (7), that is $2\sigma^4 E_o^2/l_{mn}^2$, when $\mathbf{k}_s \neq -\mathbf{k}_i$.

2.2 Double scattering by many scattereres

As is discussed above, the constructive interference always takes place in the backscattering direction. However, this holds only when two scattereres exist but is no longer valid when the number of scattereres is more than two.

Let us consider a case where the number N of particles is much larger than two. In this case there are $N(N-1)$ terms in the second sum in (1). Dividing such sum into a sum of $N(N-1)/2$ pairs, we may rewrite the double scattering field as

$$E_d(\mathbf{k}_s) \sim \sigma^2 \sum_{m,n=1, m\neq n}^{N} E_0 \frac{e^{i(\mathbf{k}_i \mathbf{r}_m + k|\mathbf{r}_m - \mathbf{r}_n| - \mathbf{k}_s \mathbf{r}_n)}}{|\mathbf{r}_m - \mathbf{r}_n|} = \sum_{pairs} \Psi^{m,n}(\mathbf{k}_s), \tag{8}$$

where *pairs* means summation over $N(N-1)/2$ pairs. Since $\Psi^{m,n}(\mathbf{k}_s)$ with different (m, n) are random in phase, the sum $E_d(\mathbf{k}_s)$ is expected to be a complex Gaussian random variable with zero average by the central limit theorem.

Let us estimate the variance of $E_d(\mathbf{k}_s)$. Since there are $N(N-1)/2$ pairs of double scattering processes, we roughly get the variance for direction far from the backscattering,

$$\langle I_2(\mathbf{k}_s)\rangle = \langle |E_d(\mathbf{k}_s)|^2\rangle \sim \frac{\sigma^4}{l^2} E_o^2 N(N-1), \tag{9}$$

and for the backscattering direction with $\mathbf{k}_s = -\mathbf{k}_i$

$$\langle I_2(-\mathbf{k}_i)\rangle = \langle |E_d(-\mathbf{k}_i)|^2\rangle \sim 2\frac{\sigma^4}{l^2} E_o^2 N(N-1). \tag{10}$$

Note that $\langle I_2(-\mathbf{k}_i)\rangle$ is twice as much as $\langle I_2(\mathbf{k}_s)\rangle$ in (9).

3. Measurement of angular distribution of the scattering

We have developed a video system measuring the light backscattered from an object in Fig. 4. The system is made up of CCD image sensor, polarizer, variable attenuater, lens with focal length $100mm$, and a laser diode emitting light beam with $\lambda = 635nm$, $P_0 = 3mW$ and spread angle 0.7 milli-radian. The half-mirror is a metal coated glass plate. The lens acts as an optical Fourier transformer, so that a speckle image $I(x, y)$ detected by CCD is proportional to the intensities of the angular distribution of the scattering from an object, and an image coordinate (x, y) physically means a pair of scattering angles. A pixel length corresponds to a scattering angle width

$$\Delta\theta = 97.2 \times 10^{-6} \text{radian}. \tag{11}$$

Therefore, the box area with 256×256 pixels in figure 2 corresponds to 24.8×24.8 milliradian in the angular distribution.

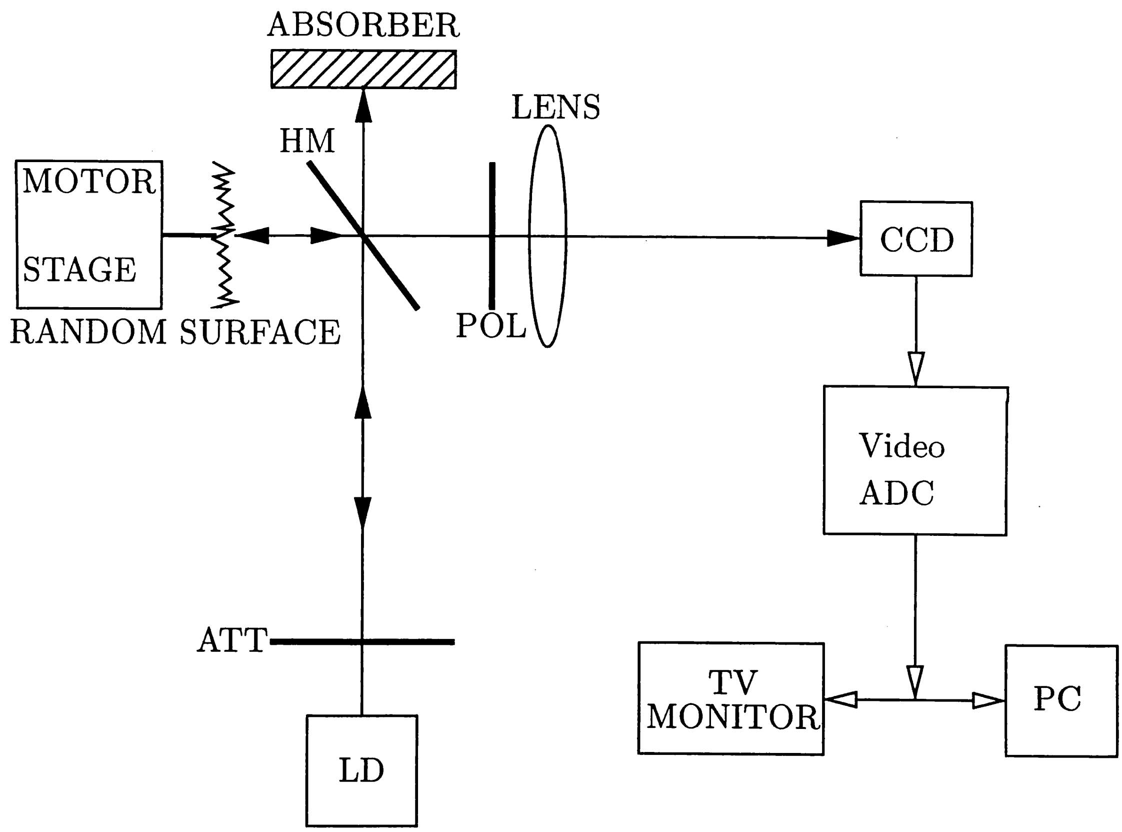

Fig. 4: System measuring the angular distribution of the scattering. LD: Laser diode emitting light beam with $\lambda = 635nm$, $P_0 = 3mW$, and spread angle $0.7m$ Radian. ATT: Variable atenuater, HM: Half-mirror, POL: Polirizer, LENS: lens with focal length $100mm$. CCD: CCD Image sensor. Digital image with 8bit levels and 640×512 pixels is obtained.

As is seen in Fig.1, the enhanced backscattering can not be observed in a sample of speckle pattern. But it can be observed by averaging many samples of speckle patterns. From engineering point of view, a question arises such that how many samples are required to clearly observe the enhanced backscattering. To answer this problem, we measure the probability distribution of light intensities, i. e., pixel values of speckle pattern. First, we divide the speckle pattern into two regions: one is the ebs region and the other the non-ebs region. The ebs region is a circular region, with a radius of 9 pixels, the center of which is located at the center of the enhanced backscattering spot, and the non-ebs region is the region excluding the ebs region. Then, we calculate the probability distributions for these regions.

Figure 5 shows the probability distributions obtained from one sample of speckle pattern scattered from a ceramic plate. In figure 5, there are no pixel values less than 25. This is probably due to DC offset in the video ADC circuit. In what follows, low pixel values less than $I_{min} = 48$ are regarded as noise caused by stray light. Here, the probability distribution for the ebs region is spiky, because the ebs region has only a few hundred pixels.

By use of the computer-controlled motor-stage, we changed illuminated portion of the ceramic surface to obtain many samples of speckle patterns. From 500 samples of speckle

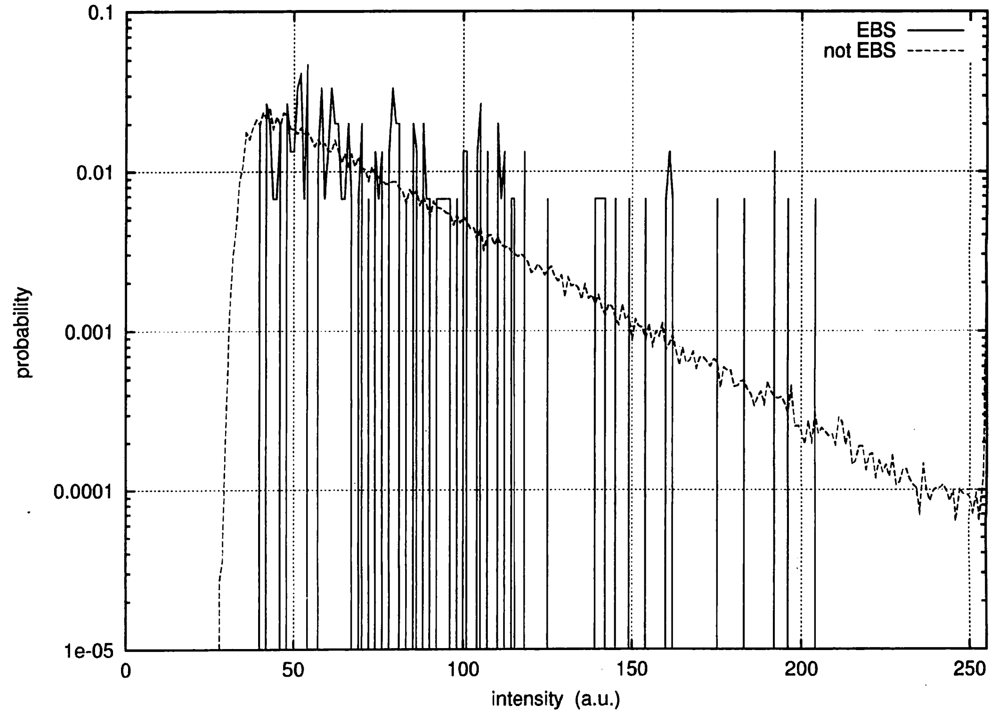

Fig. 5: Probability distribution of the intensities of the backscattering calculated from one sample of speckle pattern. Ceramics (log__cs1adE. ps). EBS: backscattering direction, not EBS: directions excluding backscattering directions. Since video ADC is 8 bit in resolution, an intensity of a pixel is in a range from 0 to 255. The probability for the ebs region is spiky, because the ebs region contains only hundreds pixels.

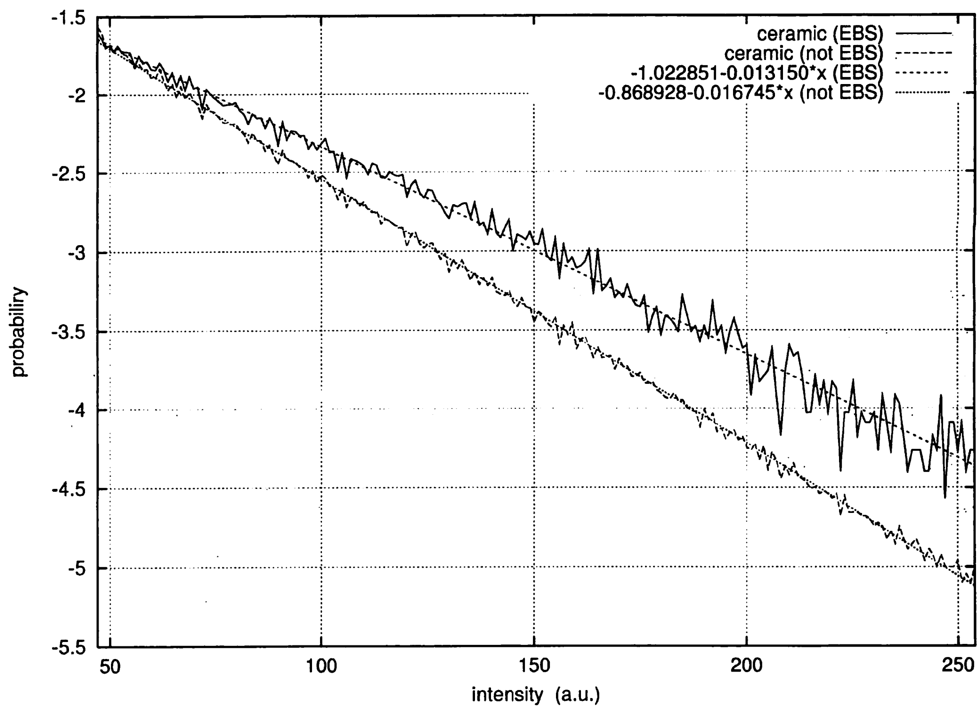

Fig. 6: Probability distribution of scattering intensities calculated from 500 samples of spackle patterns. Ceramics (kinji__ceramic500a. ps). Horizontal: intensity (arbitrary unit), Vertical: logarithm of probability. EBS: backscattering direction, not EBS: directions excluding backscattering directions. This figure shows that the probability is well approximated by the Laplace distribution.

patterns so measured, we obtain the probability distributions in Fig. 6, which can be regarded as the Laplace distribution or one-side exponential distribution in the range of $I_{min} = 48 \le I \le 255$. Assuming the distribution is exponential, we put

$$\log_{10}[P(I)] = \alpha - \beta I. \tag{12}$$

Then, we determine the parameter β by the least mean square in the range $I_{min} = 48 \le I \le 255$.

$$\beta = \begin{cases} \beta_{ebs} = 0.013150, & \text{(ebs region)} \\ \beta_{nebs} = 0.016745, & \text{(non-ebs region)} \end{cases}. \tag{13}$$

The β_{nebs} is only 27.4% larger than β_{ebs}. From (12) and (13), we can estimate the average and the standard deviation of speckle pattern,

$$\langle I - I_{min} \rangle = \frac{1}{\ln(10)\beta}, \quad \sqrt{\langle (I - \langle I \rangle)^2 \rangle} = \frac{1}{\ln(10)\beta}, \tag{14}$$

where $\beta = \beta_{ebs}$. or β_{nebs}. Because there is small difference between β_{ebs} and β_{nebs}, it is difficult to find the enhanced backscattering in a speckle pattern. To find the enhanced backscattering, we need averaging speckle patterns. Instead of average, let us consider a sum:

$$S_M(x, y) = \sum_{m=1}^{M} I_m(x, y), \tag{15}$$

where M is the number of summation, and the subscript m denotes the mth speckle pattern $I_m(x, y)$. Even when $I_m(x, y)$ is Laplace distributed random variable, $S_M(x, y)$ may be regarded as a Gaussian random variable by the central limit theorem when M is sufficiently large. The average and the standard deviation may be obtained as

$$\langle S_M - M I_{min} \rangle = \frac{M}{\ln(10)\beta}, \quad \sqrt{\langle (S_M - \langle S_M \rangle)^2 \rangle} = \frac{\sqrt{M}}{\ln(10)\beta}. \tag{16}$$

Since $S_M(x, y)$ is regarded Gaussian, we obtain an inequality

$$\frac{M}{\ln(10)\beta}\left(1 - \frac{2}{\sqrt{M}}\right) < S_M(x, y) - M I_{min} < \frac{M}{\ln(10)\beta}\left(1 + \frac{2}{\sqrt{M}}\right), \tag{17}$$

which holds with a probability 95.4%. From this, we obtain a condition to observe the enhanced backscattering,

$$\frac{M}{\ln(10)\beta_{nebs}}\left(1 + \frac{2}{\sqrt{M}}\right) \le \frac{M}{\ln(10)\beta_{ebs}}\left(1 - \frac{2}{\sqrt{M}}\right), \tag{18}$$

which means $S_M(x, y)$ for the ebs region is larger than $S_M(x, y)$ for the non-ebs region. Solving (18), we obtain the number M to observe the enhanced backscattering with a probability $1 - (1 - 0.954)^2/4 \approx 0.9989$,

$$M \ge \left(2 \frac{\beta_{ebs} + \beta_{nebs}}{\beta_{ebs} - \beta_{nebs}}\right)^2. \tag{19}$$

If we calculate the right hand side by use of (13), we find the number of samples is $M \ge 276$ to observe the backscattering enhancement.

Let us see the probability distribution of $S_M(x, y)$, which can be calculated numerically by (12) and (13). Then, we obtain the probability distribution of $S_M(x, y)$ in figure 7. From this figure, we see that there is some overlap region when $M = 50$. However, the probabilities for the ebs region and non-ebs region are well separated when $M = 200$ and 400.

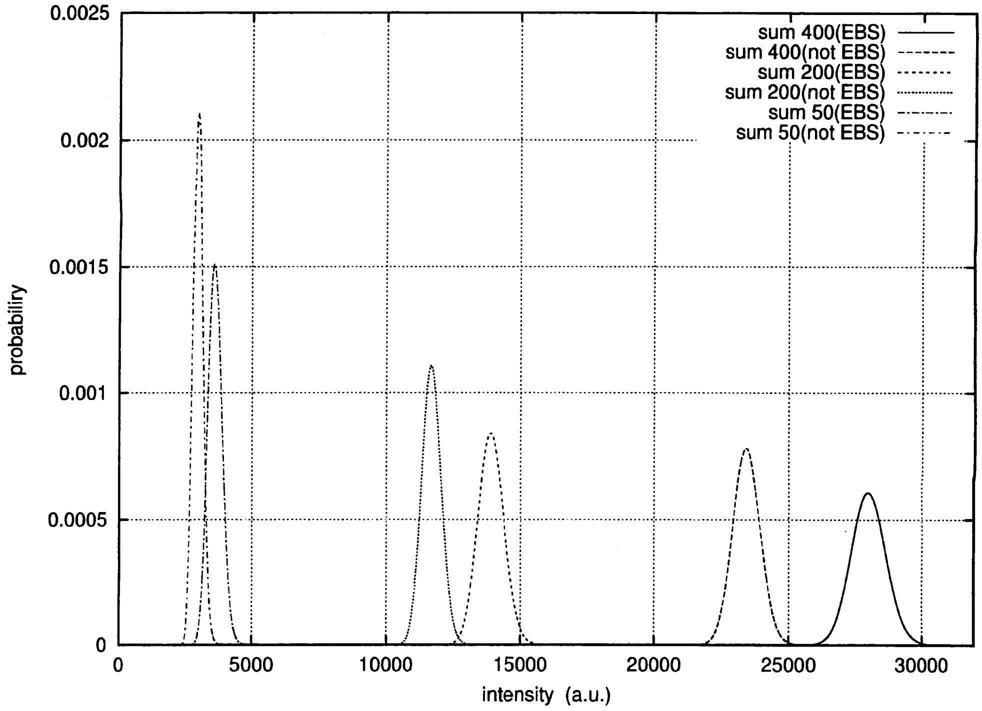

Fig. 7: Probability distribution of scattering intensities. Ceramics (kai_ceramics1. ps). sum of 50 samples of spackle patterns. sum of 200 samples of spackle patterns, and sum of 400 samples of spàckle patterns. EBS: backscattering directions, not EBS: directions excluding the backscattering directions

Therefore, we see that more than 200 samples are required to get the enhanced backscattering in this case.

From experimental observations for ceramic surfaces, we found out that the probability distribution of the scattered intensities becomes Laplace distribution, which agrees with the result (2) from theoretical considerations. Experiments were carried out for other disorder materials such as papers and polystyrene surfaces and we found again that scattered intensities have Laplace distribution.

4. Conclusions

In this report, we have discussed on the probability distribution of light intensity backscattered from a disorder system. Using a simple model of discrete scattereres distributed randomly, we pointed out that the enhanced scattering always takes place in deterministic sense when the number of scattereres is two. When the number of scattereres becomes larger than two, however, the enhanced scattering becomes a stochastic phenomenon; the probability that light intensity takes a large value becomes large in the backscattering direction. From optical experiments, we found that the probability distribution becomes Laplace distribution. In terms of the probability distribution, we discussed the number of speckle patterns to clearly observe the backscattering enhancement.

From analytical works, however, we have pointed out that the enhanced backscattering may occur for a periodic random surface[6]. We have also found that the enhanced backscattering can be caused by a combination of a single scattering and reflection[7]. Experimental study for such cases is left for future study.

Acknowledgment

This work was supported in part by the Grant-In-Aid for Scientific Research from the Ministry of Education and Science.

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