# Studies on Design of Passive Suspension System for Railway Vehicles

by

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June 2008

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A dissertation submitted in partial fulfillment of the requirements
for the degree

**Doctor of Engineering**

to

Graduate School
Kyoto Institute of Technology
Sakyo, Kyoto

June 2008

# Acknowledgements

Since my first days studying in Japan, Prof. Akira Sone, Prof. Arata Masuda and Dr. Daisuke Iba have been of great help with generous contributions to my studies. I owe them a great deal.

I would also like to thank Prof. Hiroshi Kimura and Prof. Shigeru Murata for reading and commenting on my thesis. Without their insights, this thesis wouldn't be completed.

I would also like to thank all my lab partners for their contributions to my work. Each of them offered a valuable bit of assistance for which I am truly grateful.

Finally, I would like to thank my wife for her care and encouragement. Her contribution in my education is truly worthy and does not go unnoticed.

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June 2008

# Abstract

Various kinds of railway vehicle suspensions linking the bogies and the car body have been designed to cushion riders from vibrations. In general, the suspension systems used in railway vehicles can be categorized as two main categories of suspension systems, namely passive and active suspension systems. Although active suspension systems can provide high control performance over wide frequency range of excitations induced by the rail track irregularities beyond that attainable by passive suspensions, hindered by its complexity, its cost, and its power consumption have yet to be accepted for conventional use. Therefore, passive suspension systems remain dominant in the marketplace because they are simple, reliable, and inexpensive.

This research deals with the design of passive suspension system of railway vehicles. Since the classical method that used fixed-point theory is no longer applicable to the design of passive suspension system of railway vehicle because of some its limitations such as: It is only applicable when suspension system has two degree-of-freedom, it can not apply to design a suspension system with robustness performance, and the design result often depends on designer' experience etc.. Therefore, many methods have been developed to replace it. In this study, by utilizing feedback control theories the design problem is examined from the view of feedback control problem. Consequently, the "feedback gain" is a decentralized matrix composed of the suspension parameters to be optimized. Since minimizing $\mathcal{H}_\infty$ norm of the system implies suppressing the peak of the magnitude of frequency response of the system, parameters optimization of passive suspension systems become a $\mathcal{H}_\infty$ static output feedback problem, which is solved by Bilinear-Matrix-Inequality (BMI) problem. One of the easiest methods to solve this BMI problem is alternative algorithm, which is derived from iterative schemes of alternation between analysis and synthesis via Linear Matrix Inequalities (LMIs). Thus many difficult problems in passive suspension system design become tractable in the framework of structured control. By applying this design method, the degree of freedom can be increased until our design model approaches to real-life situations and moreover we can optimize the parameters of suspension system with robustness performance in two or more states of suspension system. Three design problems corresponding to two-DOF, six-DOF and robust design are given to show the performance and computational efficiency of this new design method comparing to conventional method.

# Contents

# Chapter 1

# Introduction

## 1.1 Motivation

The development of railway vehicles has been an interest of many countries because trains have been proven as an efficient and economical transportation means. Increasing the running speed of railway vehicles is one of effective ways to make the railways more competitive with air transport while providing better safety and saving energy. However, the high speed of the train would cause significant vehicle car body vibrations, which induce the problems such as: the ride stability, the ride quality, and the cost of track maintenance. Various kinds of railway vehicle suspensions linking the bogies and the car body have been designed to cushion riders from vibrations. In general, the suspension systems used in railway vehicles can be categorized as passive, active, and semi-active types. A passive railway vehicle suspension employing springs and pneumatic or oil dampers can only store energy in the spring and dissipate energy through the damper. Both components are fixed at the design stage. If the damper is replaced with a force actuator, the suspension becomes a fully active suspension. The idea behind fully active suspensions is that the force actuator is able to apply a force to the suspension in either jounce or rebound. This force is actively governed by the sophisticated control scheme employed in the suspension, high power is also required. Nowadays, with the development of electronics and microprocessors, commercial railway vehicles with active suspensions have become available. Although active suspension systems can provide high control performance over wide frequency range of excitations induced by the rail track irregularities beyond that attainable by passive suspensions, hindered by its complexity, its cost, and its power consumption have yet to be accepted for conventional use. Therefore, passive suspension systems remain dominant in the marketplace because they are simple, reliable, and inexpensive.

The typical passive suspension system can be considered as a spring in parallel with a damper placed at each corner of the vehicle. The spring is chosen based solely on the weight of the vehicle, while the damper is the component that defines the suspension's placement on the compromise curve. Depending on the realistic condition of vehicle, a damper is chosen to make the vehicle perform best in its application. Ideally, the damper should isolate passengers from low-frequency road disturbances and absorb high-frequency road disturbances. Passengers are best isolated from low-frequency disturbances when the damping is high. However, high damping provides poor high frequency absorption. Conversely, when the damping is low, the damper offers sufficient high-frequency absorption, at the expense of low-frequency isolation.

Generally speaking, passive suspension system design was formerly used fixed-points theory [(1),(2),(3)]. This design method is based on the existence of 3 fixed-points in frequency response curves of system. By choosing the optimal positions of these 3 points, designers are able to design the optimal parameters. But this method could not be applied for complex systems that have more than 2 degree-of-freedoms and moreover the results of design usually depend on the designer's experiences. Since the classical fixed point theory is no longer applicable to the design of passive suspension system of railway vehicle, many methods have been developed to replace it.

In the recent decades, there are numerous optimization methods that have been proposed to replace the classical method which applied fixed-points theory due to some problems. Some researchers have utilized the LQG optimal control theory for the design of passive mechanical systems [(4),(5),(6),(7),(8)]. L. Zuo, et al.[(4)] and D. Iba, et al.[(3)] utilized the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norm optimization to design passive mechanical suspension [(4),(5)], MDOF tuned mass damper [(3),(6),(7)] and in vibration control of nuclear components [(8)] etc.

This study applies the $\mathcal{H}_\infty$ norm optimization to design optimal parameters of springs and dampers of railway suspension system. The purpose of this optimization is: suppressing the peaks of the magnitude of frequency response curves of suspension system at resonance, it is equivalent to imply minimizing $\mathcal{H}_\infty$ norm of our control system. Therefore, parameters optimization of passive suspension system becomes $\mathcal{H}_\infty$ static output feedback problems. In other words, the passive suspension design is equivalent to design feedback gain of a controller with structured static output. This feedback gain is generated by the springs and damping elements which need to be designed. The design problem is transformed to the Bilinear Matrix Inequality (BMI) problem, which can be solved via the alternative minimization algorithm. Thus, by applying feedback control theory, many difficult problems in passive suspension systems will become tractable in the framework of structural control and this proposed method is one of solutions to avoid limitations of conventional design method.

## 1.2 Objectives

This study focuses on three primary objectives. The first is to present an overview of design method which was applied to design passive suspension system and then establish a new design method that utilized control theory in optimizing the parameters of railway suspension. Second, two design problems corresponding to two-DOF suspension system and six-DOF suspension system are examined, these two design problems show the performance and computational efficiency of proposed design method comparing to conventional method. The third is to propose a design method of two-degree-of-freedom passive suspension system with robust performance in two states of body weight, full and empty load of body. This design is in order to provides a good operation in most common uncertain parameters of passive suspension system

## 1.3 Approach

This research succeed L. Zuo et al.'s ideal in "Design of passive mechanical systems via decentralized control theory" available at 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, AIAA2002-1282, pp.1-9. L. Zuo et al.'s ideal in "Structured $\mathcal{H}_2$ Optimization of Vehicle Suspensions Based on Multi-Wheel Models" available at Vehicle System Dynamics, Vol.40 (2003), pp. 351-371. and D. Iba et al.'s ideal in "Robust design method of multi-degree-of-freedom passive tuned mass damper" available at Proceedings of PVP2006-ICPVT-11-93364, 2006 ASME Pressure Vessels and Piping Division Conference, July 23-27, 2006

## 1.4 Outline

Chapter 1 presents the motivation for the researching in this thesis by giving a background of suspension system, then a general comparison of passive suspension system versus active suspension system and classical design method applied fixed-points theory versus new design method applied control theory are presented. The objectives and approach to this research are discussed.

Chapter 2 provides the methodology of two design methods that will be applied to design passive suspension systems along with this study. First, the classical design method using for a two-degree-of-freedom (2DOF) system will be presented. The existence of 3 fixed-points in frequency response curves of body acceleration is examined. By choosing the optimal position of these 3 points, designers are able to design the optimal parameters for suspension systems. Second, the new design method used control theory will be established, we can view problem from the view of feedback control, parameters selection and optimization of a passive suspension system becomes a control problem. Therefore, many difficult problems in passive mechanical systems become tractable in the framework of structural control. This chapter shows that applying feedback control theory in designing a passive suspension system is one of solutions to avoid limitations of classical method.

Chapter 3 describes the application of new methods in designing 2DOF passive suspension system. By giving a comparison between the results and applicable abilities of two methods, it can reveal the weak and strong points of each method.

Chapter 4, the DOF of systems will be increased to Six DOF. By investigating a passive suspension system with six-degree-of-freedom, this chapter wants to express that the degree of freedom of system can be increase until the model approaches to real-life situations. This is one of strong points of applying control theory in design passive suspension system comparing to classical design method.

Chapter 5 discusses the robust design applies to design suspension system via new method. This chapter mentions an important factor in designing an engineering system is uncertainty of some parameters, which emanates from natural randomness, limited data, or limited knowledge of systems. Concept of a good passive suspension system in this chapter is a passive suspension system that provides a good operation in most common change in vehicle weight. This chapter proposes a design method of two-degree-of-freedom passive suspension system with robust performance in two states of body weight, full and empty load of body. This chapter is confirmed to be able to design the two-degree-of-freedom passive suspension system with robustness by using control theory in particular and ability of utilizing control theory in design robustness of multi-degree-of-freedom passive suspension system in general.

Chapter 6 provides concluding remarks and a summary of the study. The purpose of this chapter is to summarize this thesis and determine weak and strong points of each method that was presented. The chapter ends with recommendations for future research in the field of railway vehicle suspensions are also discussed.

# Chapter 2

# Design Methods

This chapter provides two design methods conducted in this study. First, the classical design method using for a two-degree-of-freedom system will be presented. The existence of 3 fixed-points in frequency response curves of body acceleration is examined. By choosing the optimal position of these 3 points, designers are able to design the optimal parameters for suspension systems. Second, the new design method which utilized control theory in optimizing parameters of suspension system will be established, this new design method views design problem from the view of feedback control, therefore, parameters selection and optimization of a passive suspension system become a control problem.

## 2.1 Classical Design Method via Fixed-Points Theory

The proposed two-degree-of-freedom system has multiple masses $m_1$, $m_2$ and they are connected in series by springs $k_1$, $k_2$ and dampers $c_1$, $c_2$ as illustrated in Fig. 2.1. train wheel displacement $x_0$ takes the role of excitation. $x_1$, $x_2$ are vertical translational motions of bogie and body respectively.

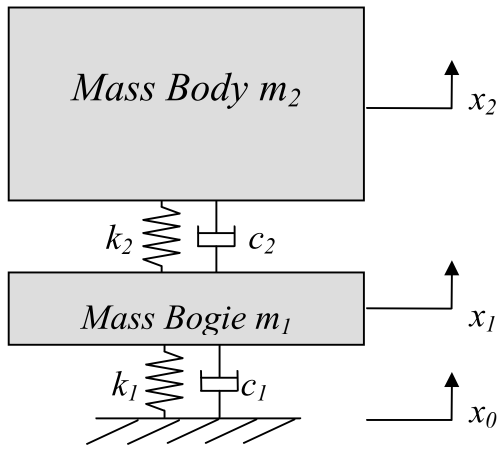


**Fig. 2.1. Two-DOF suspension system model**

The equations of motion can be written as:

$$m_1\ddot{x}_1 + k_1x_1 + c_1\dot{x}_1 = -k_2x_1 + k_2x_2 - c_2\dot{x}_1 + c_2\dot{x}_2 + k_1x_0 + c_1\dot{x}_0$$

$$m_2\ddot{x}_2 = k_2x_1 - k_2x_2 + c_2\dot{x}_1 - c_2\dot{x}_2 \quad (2.1)$$

where $x_0$ is displacement which is generated by rail tracks error: $x_0 = a_0\sin\omega t$.

General speaking, a good suspension should provide a comfortable ride and good handling within a reasonable range of deflection. Moreover, these criteria subjectively depend on the purpose of the vehicle. For example, a freight vehicle driver will accept a relatively hard ride as a compromise for high speed handling and safe fast cornering. But the same ride would be intolerable for the passengers of a Pullman vehicle.

Normally, the stiffness of springs is chosen based solely on the weight of the vehicle, while the damper is the component that defines the suspension's placement on the classic suspension compromise curve in Fig. 2.2. Ideally, the damper should isolate passengers from low-frequency tracks disturbances and absorb high-frequency tracks disturbances. Passengers are best isolated from low-frequency disturbances when the damping is high. However, high damping provides poor high frequency absorption. Conversely, when the damping is low, the damper offers sufficient high-frequency absorption, at the expense of low-frequency isolation.

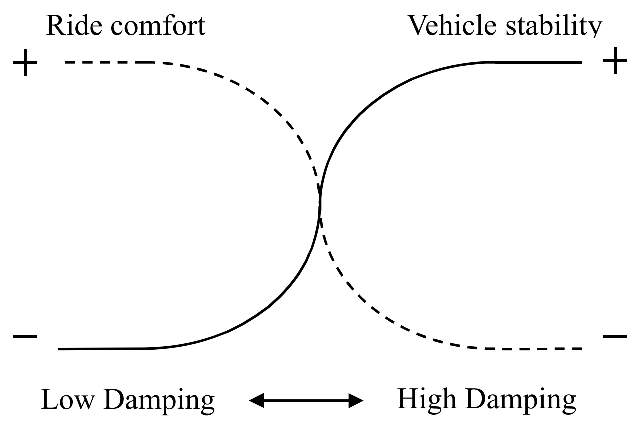


**Fig. 2.2. Relationship between ride comfort and vehicle stability**

If we just consider how to reduce the vibration of vehicle's body, we should choose the spring as soft as possible. However, it will be dangerous when driving. In designing passive suspension of commercial railway vehicles, the stiffness of primary $k_1$ can be defined roughly 2～2.5MN/m for 1 axle (15). $c_1$, $c_2$, $k_2$ are parameters which need to be designed.

Given parameters of scale model of railway vehicle are shown in Table 2.1.

**Table 2.1 Given parameters of railway vehicle**

| | | |
|---|---|---|
| $m_1$ | 3,000 | Kg |
| $m_2$ | 20,000 | Kg |
| $k_1$ | 2,200,000 | N/m |

By applying fixed-points theory we can choose optimally values of $c_1$ and $k_2$, this method is based on the existence of three fixed-points in frequency response curves of body displacement. To exist three fixed-points in frequency response curves, designer have to consider which dampers are main dampers then neglect auxiliary dampers (2).

In our case, $m_1 << m_2$ hence, $c_1$ is chosen as damping of main dampers and $c_2$ is damping of auxiliary dampers. For this reason we can neglect auxiliary dampers, thus damping of auxiliary dampers $c_2 = 0$ then $\varepsilon_2 = 0$ ($\varepsilon_2 = \dfrac{c_2}{m_2 \nu_1}$)

By solving the equation of motion (2.1) with $\varepsilon_2 = 0$, acceleration of vehicle body can be defined as following equation [(2)].

$$\left(\frac{a\omega^2}{a_0 \nu_1^2}\right)^2 = \frac{(1+\varepsilon_1^2\eta^2)\kappa^2\eta^4}{[\mu\eta^4 - \{1+(1+\mu)\kappa\}\eta^2 + \kappa]^2 + \varepsilon_1^2(\kappa-\eta^2)^2\eta^2} \tag{2.2}$$

where $a\omega^2$ is the acceleration of vehicle body, $a$ is the displacement of vehicle body, $a_0$ is maximum amplitude of excitation $x_0$ and

$$\mu = \frac{m_1}{m_2}; \kappa = \frac{k_2}{k_1}; \varepsilon_1 = \frac{c_1}{m_2 \nu_1}; \eta = \frac{\omega}{\nu_1}; \nu_1 = \sqrt{\frac{k_1}{m_2}}$$

From Eq. (2.2) we can plot the frequency response curves of body acceleration, as shown in Fig. 2.3. There are three fixed-points P1, Q and P2 and all curves will go through these three fixed-points.

If ratio of $k_2$ to $k_1$ increases, then altitude of point P1 will be decreased and at Q will be increased. Therefore, the optimal value of ratio of $k_2$ to $k_1$ will exist when the altitudes of these two points are the same level as shown in Fig. 2.4.

To get optimally the value of $\varepsilon_1$, we choose the mean value of $\varepsilon_1$ so that curve reaches maximum altitude at P1 and Q. From the optimal values of $\kappa$ and $\varepsilon_1$ we can define the parameters of $k_2$ and $c_1$

According to condition of minimizing the body frequency response curve at frequency of Q when the system doesn't have auxiliary damper [(2)], we can define the damping coefficient of secondary damper $c_2$ by following equation:

$$\varepsilon_2 = \frac{1}{2\varepsilon_1}\left[\mu\left\{1-\frac{1}{(1+\mu)\kappa}\right\}\left\{(1+\mu)\kappa+\frac{1-\mu}{1+\mu}\right\}-\frac{1-\mu}{1+\mu}\varepsilon_1^2+\sqrt{\left[\mu\left\{1-\frac{1}{(1+\mu)\kappa}\right\}\left\{(1+\mu)\kappa+\frac{1-\mu}{1+\mu}\right\}-\frac{1-\mu}{1+\mu}\varepsilon_1^2\right]^2+\frac{4\mu\kappa}{1+\mu}\varepsilon_1^2}\right]$$

with $\varepsilon_2 = \dfrac{c_2}{m_2 \nu_1}$

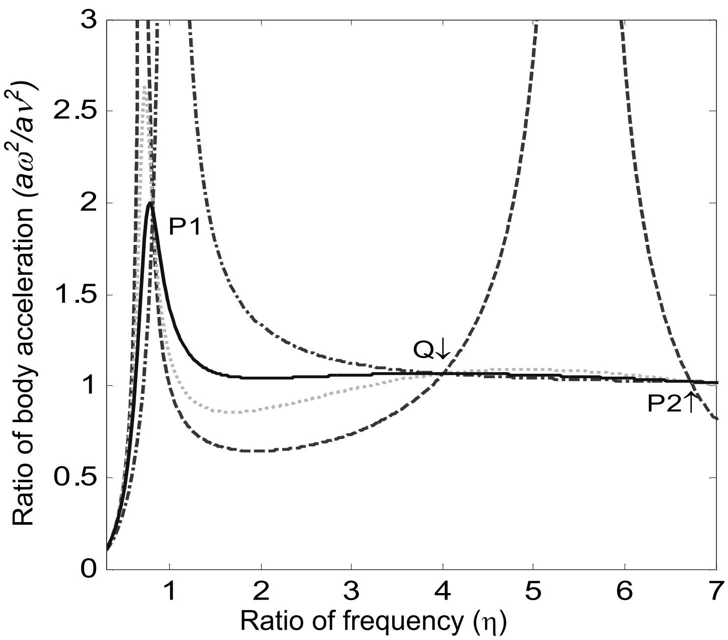


**Fig. 2.3. Three fixed points in frequency response curves**

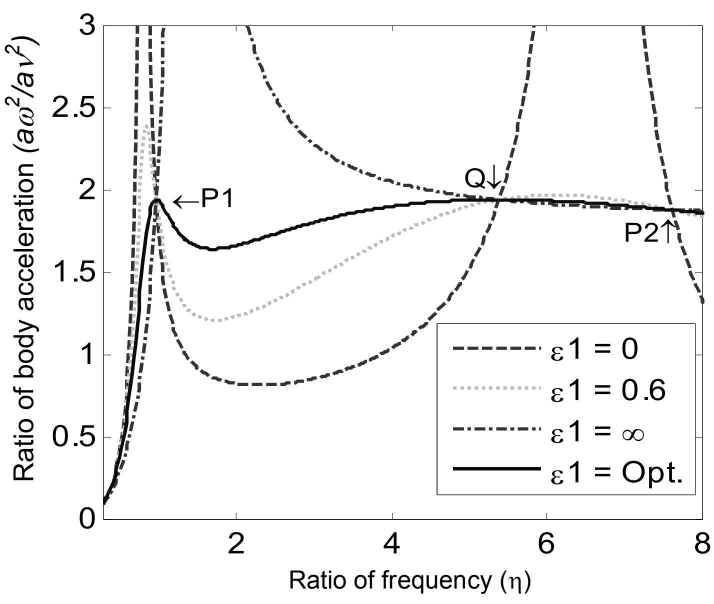


**Fig. 2.4. Frequency response curves with optimal value of** $\kappa$

By choosing the optimal positions of these three points, designers are able to design optimal parameters of suspension system.

Designed results by using fixed-points theory are shown in Table 2.2.

**Table 2.2 Design results by using fixed-points theory**

| | | |
|---|---|---|
| $c_1$ | 216,960 | Ns/m |
| $c_2$ | 52,906 | Ns/m |
| $k_2$ | 3,576,600 | N/m |

# 2.2 Design Problem in the View of Control Theory

This section examines the design of a passive suspension in the view of designing a controller with structured static output feedback. The feedback gains are generated by the springs and damping elements which need to be designed. The closed-loop inputs are the track disturbances and the closed-loop outputs are the performance indices.

## 2.2.1 Problem Formulations

By examining the design problem from the view of feedback control, the springs feedback the relative displacements locally, the damping elements feedback the relative velocities locally, and the control forces are generated by springs and dampers which need to be designed [(4)].

The equations of motion in matrix form can be written as:

$$M\ddot{X} + KX + C\dot{X} = Eu + F_p f + F_v \dot{f} \tag{2.3}$$

where: *M, K, C* are positive definite equivalent mass, stiffness and damping matrices respectively, *u* is control force which is generated by the springs and damping elements to be designed, $f$ and $\dot{f}$ are vectors of displacement excitations and velocity excitations, $F_p$ and $F_v$ are positive definite stiffness of equivalent springs and damping of equivalent dampers matrices which connect between our suspension system and excitation base

By defining the state variable as:

$$x_s = \begin{bmatrix} X & \dot{X} - M^{-1}F_v f \end{bmatrix}^T \tag{2.4}$$

The equation of motion can be written in state-space form as:

$$\dot{x}_s = Ax_s + B_1 f + B_2 u \tag{2.5}$$

where $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}; B_1 = \begin{bmatrix} M^{-1}F_v \\ M^{-1}\left(F_p - CM^{-1}F_v\right) \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix};$

The vector of "measured" output Y can be written as:

$$Y = C_2 x_s + D_{21} f \tag{2.6}$$

We can write the vertical displacement of train body as an output vector *z,* which can be expressed in the form:

$$z = C_1 x_s + D_{11} f \tag{2.7}$$

where $C_1 = GC_2; D_{11} = GD_{21};$

The forces generated by the suspension springs and dampers are determined from $Y$ according to: $u = K_S Y$ where the "feedback gain" $K_s$ is a decentralized matrix (block-diagonal) composed of the suspension parameters to be optimized.

Equations (2.5), (2.6) and (2.7) cast the design of suspension system of train as a decentralized control problem, as indicated by the diagram shown in Fig. 2.5. Based on this formulation, we use decentralized control techniques $\mathcal{H}_\infty$ to directly optimize the stiffness and damping coefficients of springs and dampers to achieve performance (measured by $z$) under the disturbance of $f$. The goal of solving this problem is to determine the feedback law:

$$u = K_S Y \tag{2.8}$$

The feedback gain $K_s$ is a decentralized matrix composed of the parameters to be designed, and all parameters of springs and dampers which need to be designed are designable by determining $K_s$.

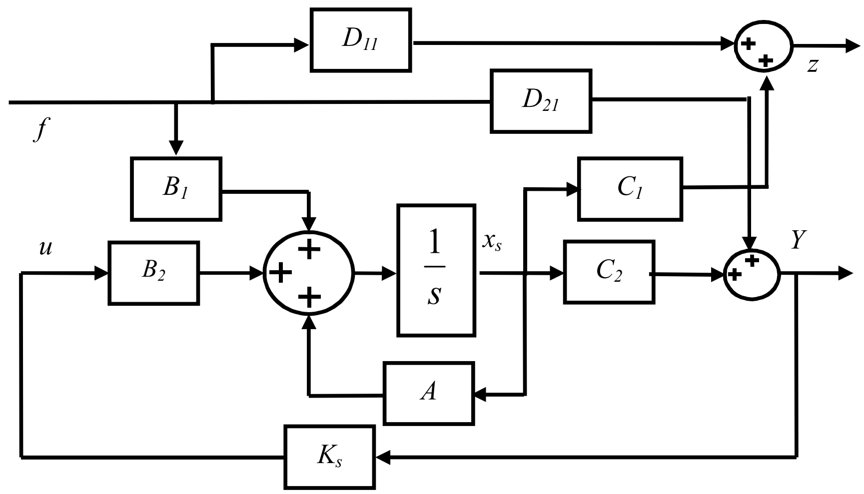


**Fig. 2.5. Block diagram of feedback control system**

## 2.2.2 $\mathcal{H}_\infty$ Control Theory and $\mathcal{H}_\infty$ Control Problem Based on BMI

The $\mathcal{H}_\infty$ control problem is defined as follows [10].

Definition 1: Given a scalar $\gamma > 0$. The controller $K_s$ is a "$\mathcal{H}_\infty$ controller", if two following conditions are met:

- The closed loop system is asymptotically stable.
- $\left\|G_{zf}\right\|_\infty < \gamma$

where $\left\|G_{zf}\right\|_\infty$ is the $\mathcal{H}_\infty$ norm (the maximum gain from $f$ to $z$) of the General Plant (Fig. 2.6).

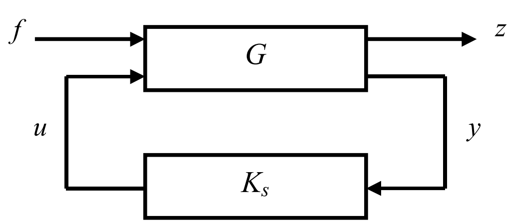


**Fig. 2.6. General plant**

The $\mathcal{H}_\infty$ norm of an LMI system $H_{zf}$ is the gain of input energy to output energy. For multi-input-multi-output linear time invariant systems, it is the supreme of the largest singular value over all frequencies [4]. Minimizing the $\mathcal{H}_\infty$ norm of the system is equivalent to suppressing the peak of the magnitude of the frequency response.

$$\begin{aligned}\left\|H_{zf}\right\|_\infty^2 &= \sup_{\omega\in R}\sigma_{\max}^2(H(j\omega))\\ &= \sup_{\omega\in R}\lambda_{\max}(H'(j\omega)H(j\omega))\end{aligned}$$

The following theorem is a link between $\mathcal{H}_\infty$ optimization and LMIs [4].

**Theorem:** The continuous-time LMI system (*A, B, C, D*) is stable and the $L_2$ gain is less than $\gamma$, if and only if there exists some symmetric matrix $X$ such that:

$$\begin{bmatrix} A'X+XA & XB & C' \\ B'X & -\gamma I & D' \\ C & D & -\gamma I \end{bmatrix} < 0$$

$$X > 0$$

From Eqs. (2.5), (2.6) and (2.7), the generalized plant is derived as:

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_2 & D_{21} & 0 \\ C_1 & D_{11} & 0 \end{bmatrix}$$

The closed-loop system from $f$ to $z$ can be written in the standard form:

$$\begin{aligned}\dot{x}_s &= \Psi x_s + \Phi f\\ z &= \Theta x_s + \Gamma f\end{aligned}$$

where the closed-loop system is given by

$$\left[\begin{array}{c|c} \Psi & \Phi \\ \hline \Theta & \Gamma \end{array}\right] = \left[\begin{array}{c|c} A+B_2K_SC_2 & B_1+B_2K_SD_{21} \\ \hline C_1 & D_{11} \end{array}\right]$$

Substituting the closed-loop system into above theorem, our problem is expressed as follows:

The continuous-time LMI system ($\Psi$, $\Phi$, $\Theta$, $\Gamma$) is stable and the $K_s$ gain is less than $\gamma$ if and only if there exists some symmetric matrix $S$ such that:

$$\begin{bmatrix} \Psi' S + S\Psi & S\Phi & \Theta' \\ \Phi' S & -\gamma I & \Gamma' \\ \Theta & \Gamma & -\gamma I \end{bmatrix} < 0$$

$$S = S' > 0 \qquad (2.9)$$

$$K_s \geq 0$$

Static decentralized control turns out to be a BMI problem. Generally BMI problems are not convex and have multiple local solutions [11] and it is not solvable in polynomial time. Hence, solving BMI problems is more complex than Linear Matrix Inequality (LMI). Many methods were proposed to search for the local minima of BMI. The easiest method to implement for solving BMI problem that is alternative minimization algorithm, this method is based on iterative schemes of alternation between analysis and synthesis via LMIs. It will transform the BMI problem to LMI problems, which can be solved easily via LMI solver [12],[13]. However, this algorithm might converge very slowly and even stop at a non-stationary point. The choice of initial values is important for convergence so that an acceptable solution could be achieved.

### Alternative Algorithm

Starting with a stabilized $K_s$ and repeat OP1 and OP2 until $\gamma$ can no longer decrease [3] in the alternative algorithm.

OP1: Fix $K_s$, minimize $\gamma$ over the $S$, subject to constraints (2.9).

OP2: Fix $S$, minimize $\gamma$ over the $K_s$, subject to constraints (2.9).

Since $K_s$ and $S$ are fixed in Eq. (2.9) and OP1 and OP2 become LMI problems, it can be solved easily by LMI solver. The alternative minimization will generate a decreasing sequence of $\gamma$, and it works well in most practical problems.

## 2.3 Summary

This chapter provides two methods of design a two-degree-of-freedom passive suspension system: a classical method which utilities fixed-points theory in optimizing parameters and a new method which utilities control theory in optimizing parameters.

In the beginning of this chapter, author presented classical design method via fixed-points theory. This section explained to readers some limitations of classical design method such as: it is complicated, the design results usually depend on the designer's experiences and this method could not be applied for complex systems that have more than 2 degree-of-freedoms. Since the classical fixed-points theory is no longer applicable to the design of a passive suspension system, other design methods are necessary to develop and replace it. In the ending of this chapter a new design method via control theory is established, passive suspension design is equivalent to design of a controller with decentralized architecture and additional constrains on the symmetry of the vehicle and the ranges of the design parameters, therefore, many difficult problems in passive mechanical systems become tractable in the framework of structural control. This chapter reveals that applying control theory in designing a passive suspension system is one of solutions to avoid foregoing limitations of classical method.

# Chapter 3

# Two-DOF Passive Suspension System via Control Theory

This chapter provides the design result of two design methods conducted in this study in the case the system has two-degree of freedom. By giving a comparison between the results of two methods, this chapter will express the weak and strong points of two mentioned design method.

## 3.1 Problem Formulations

Equation (2.1) can be written as:

$$M\ddot{X} + KX + C\dot{X} = Eu + F_p x_0 + F_v \dot{x}_0 \tag{3.1}$$

where: *M, K* and *C* are positive definite mass, stiffness and damping matrices respectively, *u* is control force generated by the springs and damping elements which need to be designed, $k_1$ is considered as a given parameter and

$$M=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix};\ X=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix};\ K=\begin{bmatrix} k_1 & 0 \\ 0 & 0 \end{bmatrix};\ C=\begin{bmatrix} c_1 & 0 \\ 0 & 0 \end{bmatrix};\ F_p=\begin{bmatrix} k_1 \\ 0 \end{bmatrix};\ F_v=\begin{bmatrix} c_1 \\ 0 \end{bmatrix};\ u=K_sY;$$

$$E=\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix};\ K_S=\mathrm{diag}\left(k_2 \quad k_2 \quad c_2 \quad c_2\right);\ Y=\left[x_1 \quad x_2 \quad \dot{x}_1 \quad \dot{x}_2\right]^T;$$

By defining the state variable as:

$$x_S=\left[X \quad \dot{X}-M^{-1}F_vx_0\right]^T.$$

The equation of motion can be written in state-space form as:

$$\dot{x}_S=Ax_S+B_1x_0+B_2u \tag{3.2}$$

where $A=\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix};B_1=\begin{bmatrix} M^{-1}F_v \\ M^{-1}\left(F_p-CM^{-1}F_v\right) \end{bmatrix};B_2=\begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix}$

Based on the geometry of the train model, we write the vector of "measured" outputs, the relative displacements and velocities at the suspension connections as a linear combination of the states and inputs, that is:

$$Y=C_2x_S+D_{21}x_0 \tag{3.3}$$

where $C_2=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ D_{21}=\begin{bmatrix} 0 \\ 0 \\ T_1M^{-1}F_v \\ T_2M^{-1}F_v \end{bmatrix};\ T_1=\begin{bmatrix} 1 & 0 \end{bmatrix};\ T_2=\begin{bmatrix} 0 & 1 \end{bmatrix};$

The vertical displacement of train body $x_2$ is output vector $z$, which can be expressed in the form:

$$z=C_1x_S+D_{11}x_0 \tag{3.4}$$

where $C_1=NC_2;D_{11}=ND_{21};\ N=\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$

The forces generated by the suspension springs and dampers are determined from $Y$ according to: $u=K_SY$ where the "feedback gain" $K_s$ is a decentralized matrix (block-diagonal) composed of the suspension parameters to be optimized. In this problem $K_s$ is shown as:

$$K_s = \mathrm{diag}\begin{pmatrix} k_2 & k_2 & c_2 & c_2 \end{pmatrix} \tag{3.5}$$

Equations (3.2), (3.3) and (3.4) cast the design of the suspension system of train as a decentralized control problem, as indicated by the diagram [9] shown in Fig. 2.4. Based on this formulation, we use decentralized control techniques $\mathcal{H}_\infty$ to directly optimize the stiffness and damping coefficients of springs and dampers to achieve performance (measured by $z$) under the excitation of $x_0$. The goal of solving this problem is to determine the feedback law:

$$u = K_s Y \tag{3.6}$$

The feedback gain $K_s$ is a decentralized matrix composed of the parameters to be designed, and all parameters of springs and dampers which need to be designed are designable by determining $K_s$.

## 3.2 Design of Two-DOF Passive Suspension System of Railway Vehicle

Given parameters of scale model of railway vehicle are shown in Table 3.1. Parameters of primary stage of railway suspension system are determined by classical method.

**Table 3.1 Given parameters of railway vehicle**

| | | | | | |
|---|---|---|---|---|---|
| $m_1$ | 3,000 | (Kg) | $m_2$ | 20,000 | (Kg) |
| $k_1$ | 2,200,000 | (N/m) | $c_1$ | 216,960 | (Ns/m) |

To solve BMI problems by alternative algorithm, we have chosen initial values for the alternative algorithm. In this design problem, the initial values are the parameters which are optimized by using fixed point theory (Table 2.2).

Start with a stabilizing $K_s$ and repeat OP1 and OP2 until $\gamma$ can not decrease any more then by the results of obtained iterations when $\gamma$ does not decrease, we obtain the final parameters. Final design parameters are shown in Table 3.2.

**Table 3.2 Final design parameters**

| | | | | | |
|---|---|---|---|---|---|
| $k_2$ | 2,181,700 | (N/m) | $c_2$ | 237,660 | (Ns/m) |

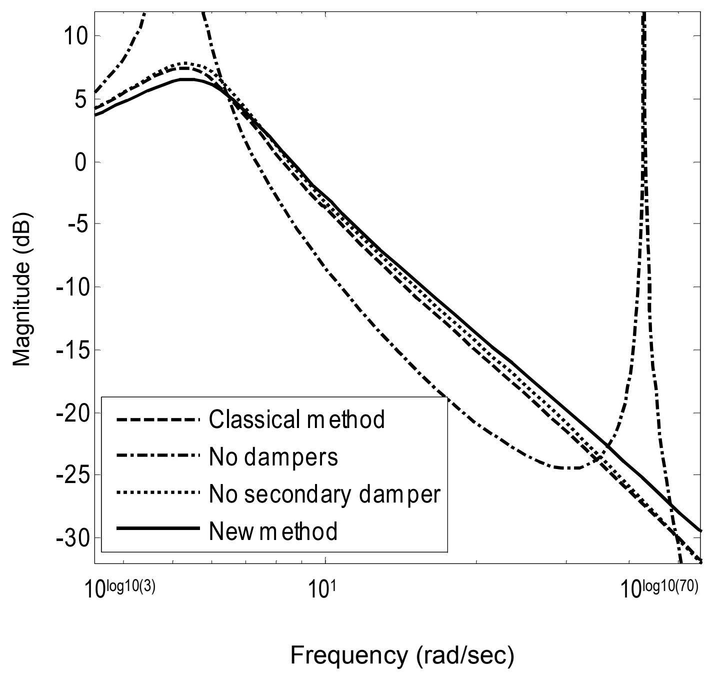


**Fig. 3.1. Frequency responses of suspension system**

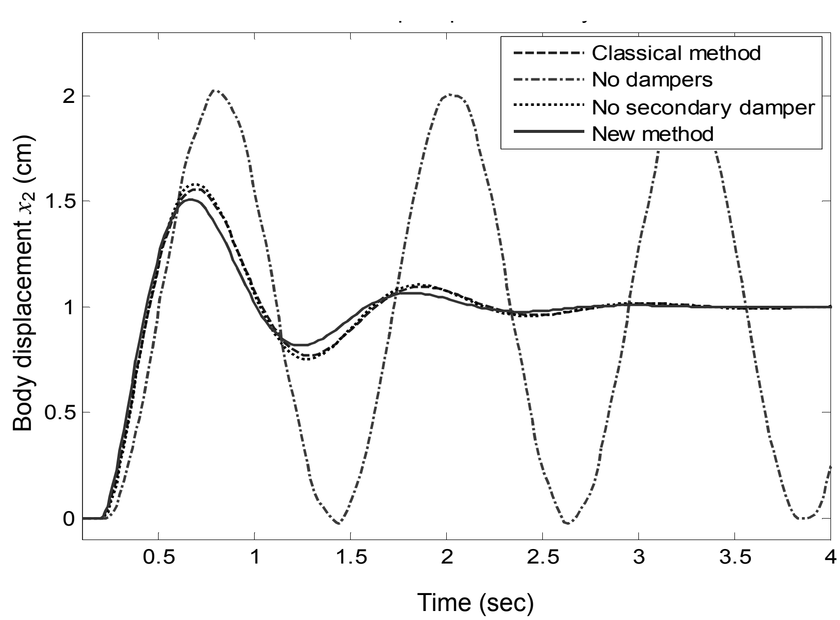


**Fig. 3.2. Unit-step response of train**

Figures 3.1, 3.2 show frequency responses and unit-step responses of train body when suspension system were designed by classical method and new method.

The dash-curve is the frequency response of system with parameters which were designed by using classical method (fixed-points theory).

Solid-curve is the gain of frequency response of system with final parameters which was designed by decentralized $\mathcal{H}_\infty$ optimization.

Dot-curve is the frequency response of system with parameters which were designed by using classical method in the case no damping in secondary stage of railway suspension system.

Dash dot-curve shows the frequency response of suspension system in the case of no dampers in both stage of railway suspension system.

It can be observed that the peak of solid-curve is the lowest peak comparing to other curves, Fig. 3.2 also demonstrates that the suspension system designed by decentralized $\mathcal{H}_\infty$ optimization can absorb vibration energy faster than classical method.

From these figures, we can see that the performance of suspension system which was designed by control theory is significantly better than that of the designed system by classical method.

## 3.3 Summary

This chapter presented and compared the design results of two design methods mentioned in chapter 2, when our passive suspension system has two-degree of freedom.

In new design method, design problem turns into BMI problem and it was solved by using alternative algorithm. The initial values for alternative algorithm are the values of parameters which are optimized by using fixed point theory. It has been clear that the results of design by using classical method are quite close to optimal parameters. Thus, this algorithm converges quickly.

This chapter has been shown that the peaks of frequency response curve which was plotted by using result of new method are lowest at resonance frequency. It proved that the result of design by using control theory was better than classical method. In other words, the performance of this system which was designed by control theory is significantly better than that of the designed system by classical method.

# Chapter 4
# Six-DOF Passive Suspension System

This chapter proposes a design method of six-degree-of-freedom passive suspension system by utilizing feedback control theory mentioned in chapter 2. By investigating a passive suspension system with six-degree-of-freedom, this chapter wants to express that the degree of freedom of system can be increased until the model approaches to real-life situations. This is one of strong points of applying control theory in design passive suspension system compared to classical design method.

## 4.1 Problem Formulations

The design problem of the proposed six-degree-of-freedom system has multiple masses $m_0$, $m_1$, $m_2$ and they are connected in series by springs $k_0$, $k_1$, $k_2$ and dampers $c_0$, $c_1$, $c_2$ as illustrated in Fig. 4.1. $x_{tr1}$ and $x_{tr2}$ are displacements of rail tracks excitations, $l$ is standard distance between rail tracks. In this model, the motion of train body, bogie and wheel-axles can be simultaneously translational and rotational in two-dimensional space. $x_{01}$, $x_{02}$, $x_1$, $x_2$, $x_3$, and $x_4$ are translational motions at wheels, at sides of bogie and side of body respectively. $J_0$, $J_1$ and $J_2$ are inertia moments of wheel-set, bogie and body respectively.

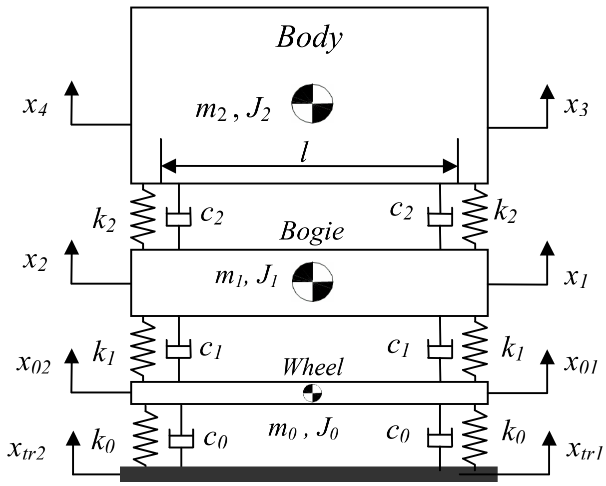


**Fig. 4.1. Six-DOF suspension system model of train**

By examining the design problem from the view of feedback control, the springs feed back the relative displacements locally, the damping elements feed back the relative velocities locally, and the control forces are generated by springs and dampers which need to be designed [4]. From this point, the equations of motion can be written as:

$$M\ddot{X} + KX + C\dot{X} = Eu + F_p f + F_v \dot{f} \tag{4.1}$$

where $M$, $K$, $C$ are positive definite equivalent mass, stiffness and damping matrices respectively, $u$ is control force generated by springs and dampers which need to be designed, $f$ and $\dot{f}$ are vectors of displacement excitations and velocity excitations and

$$K = \begin{bmatrix} k_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; C = \begin{bmatrix} c_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; F_p = \begin{bmatrix} k_0 & 0 \\ 0 & k_0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; F_v = \begin{bmatrix} c_0 & 0 \\ 0 & c_0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$
M = \begin{bmatrix}
\frac{J_0 + m_0 \frac{l^2}{4}}{l^2} & \frac{-J_0 + m_0 \frac{l^2}{4}}{l^2} & 0 & 0 & 0 & 0 \\
\frac{-J_0 + m_0 \frac{l^2}{4}}{l^2} & \frac{J_0 + m_0 \frac{l^2}{4}}{l^2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{J_1 + m_1 \frac{l^2}{4}}{l^2} & \frac{-J_1 + m_1 \frac{l^2}{4}}{l^2} & 0 & 0 \\
0 & 0 & \frac{-J_1 + m_1 \frac{l^2}{4}}{l^2} & \frac{J_1 + m_1 \frac{l^2}{4}}{l^2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{J_2 + m_2 \frac{l^2}{4}}{l^2} & \frac{-J_2 + m_2 \frac{l^2}{4}}{l^2} \\
0 & 0 & 0 & 0 & \frac{-J_2 + m_2 \frac{l^2}{4}}{l^2} & \frac{J_2 + m_2 \frac{l^2}{4}}{l^2}
\end{bmatrix};
$$

$$
E = \begin{bmatrix}
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1
\end{bmatrix};
$$

$$
f = \begin{bmatrix} x_{tr1} & x_{tr2} \end{bmatrix}^T ; u = K_S Y \; ; X = \begin{bmatrix} x_{01} & x_{02} & x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T ;
$$

$$
Y = \begin{bmatrix} x_{01} & x_{02} & x_1 & x_2 & x_1 & x_2 & x_3 & x_4 & \dot{x}_{01} & \dot{x}_{02} & \dot{x}_1 & \dot{x}_2 & \dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \dot{x}_4 \end{bmatrix}^T ;
$$

By defining the state variable as:

$$
x_s = \begin{bmatrix} X & \dot{X} - M^{-1} F_v f \end{bmatrix}^T \qquad (4.2)
$$

The equation of motion can be written in state-space form as:

$$
\dot{x}_s = A x_s + B_1 f + B_2 u \qquad (4.3)
$$

where $A=\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}; B_1=\begin{bmatrix} M^{-1}F_v \\ M^{-1}\left(F_p-CM^{-1}F_v\right) \end{bmatrix}; B_2=\begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix}$

The vector of "measured" output $Y$ can be written as:

$$Y = C_2 x_S + D_{21} f \tag{4.4}$$

We can write the vertical displacement of train body ($x_3$ and $x_4$) as an output vector $z$, which can be expressed in the form:

$$z=\left[x_3 \quad x_4\right]^T = C_1 x_S + D_{11} f \tag{4.5}$$

where:

$$C_1 = GC_2; D_{11} = GD_{21};\ G=\begin{bmatrix} 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \end{bmatrix}$$

The forces generated by the suspension springs and dampers are determined from $Y$ according to: $u = K_S Y$ where the "feedback gain" $K_s$ is a decentralized matrix (block-diagonal) composed of the suspension parameters to be optimized. In this problem $K_s$ is shown as:

$$K_S = \text{diag}\left(k_1 \quad k_1 \quad k_1 \quad k_1 \quad k_2 \quad k_2 \quad k_2 \quad k_2 \quad c_1 \quad c_1 \quad c_1 \quad c_1 \quad c_2 \quad c_2 \quad c_2 \quad c_2\right)$$

Equations (4.3), (4.4) and (4.5) cast the design of the suspension system of train as a decentralized control problem, as indicated by the diagram [9] shown in Fig. 2.4.

Based on these formulations, we use decentralized control techniques $\mathcal{H}_\infty$ to directly optimize the stiffness and damping coefficients of springs and dampers to achieve performance (measured by $z$) under the disturbance of $f$. The goal of solving this problem is to determine the feedback law:

$$u = K_s Y \tag{4.6}$$

The feedback gain $K_s$ is a decentralized matrix composed of the parameters to be designed, and all parameters of springs and dampers which need to be designed are designable by determining $K_s$.

## 4.2 Design of Six-DOF Passive Suspension System of Railway Vehicle

Given parameters of scale model of railway vehicle are shown in Table 4.1.

**Table 4.1 Given parameters of scale model of railway vehicle**

| | | | | | |
|---|---|---|---|---|---|
| $m_0$ | 3,500 | (Kg) | $J_0$ | 5,000 | (Kg·m$^2$) |
| $m_1$ | 3,000 | (Kg) | $J_1$ | 4,000 | (Kg·m$^2$) |
| $m_2$ | 20,000 | (Kg) | $J_2$ | 60,000 | (Kg·m$^2$) |
| $c_0$ | 100,000,000 | (Ns/m) | $k_0$ | 1,000,000,000 | (N/m) |

To solve BMI problems by alternative algorithm, we have chosen initial values for the alternative algorithm. Initial values for alternative algorithm are shown in Table 4.2.

**Table 4.2 Initial values for alternative algorithm**

| | | | | | |
|---|---|---|---|---|---|
| $k_1$ | 2,200,000 | N/m | $c_1$ | 216,960 | Ns/m |
| $k_2$ | 3,576,600 | N/m | $c_2$ | 52,906 | Ns/m |

Start with a stabilizing $K_s$ and repeat OP1 and OP2 until $\gamma$ can not decrease any more then by the results of obtained iterations when $\gamma$ do not decrease, we obtain the final parameters [14].

Final design parameters are shown in Table 4.3.

**Table 4.3 Final design parameters**

| | | | | | |
|---|---|---|---|---|---|
| $k_1$ | 1,783,000 | N/m | $c_1$ | 217,000 | Ns/m |
| $k_2$ | 1,614,800 | N/m | $c_2$ | 45,200 | Ns/m |

Figures 4.2, 4.3, 4.4 and 4.5 show four frequency response outputs corresponding to two inputs respectively. The dash curves are the frequency response of system with parameters which were designed by using classical method (fixed-point theory). Solid curves are the gain of frequency response of system with final parameters which was designed by decentralized $\mathcal{H}_\infty$ optimization. Dot curves are the gain of frequency response

of system in the case of system has no dampers in both stages.

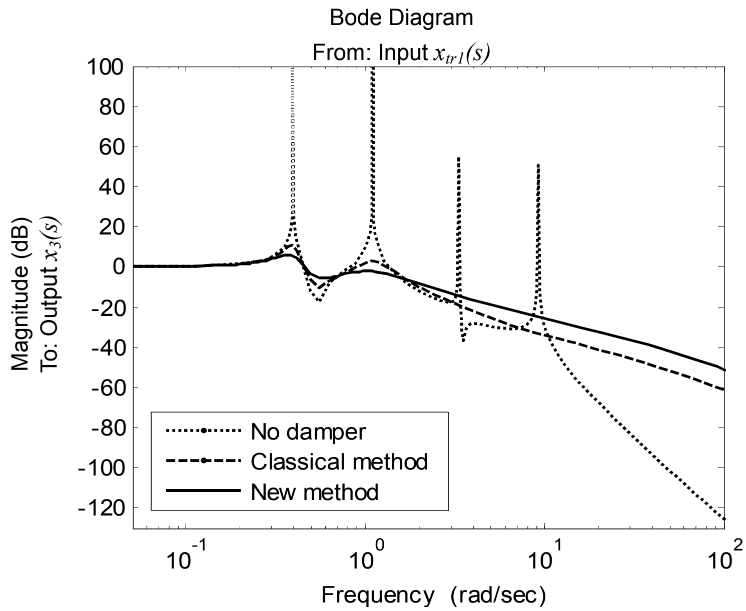


**Fig. 4.2. Gain of frequency response of $x_3(s)/x_{tr1}(s)$**

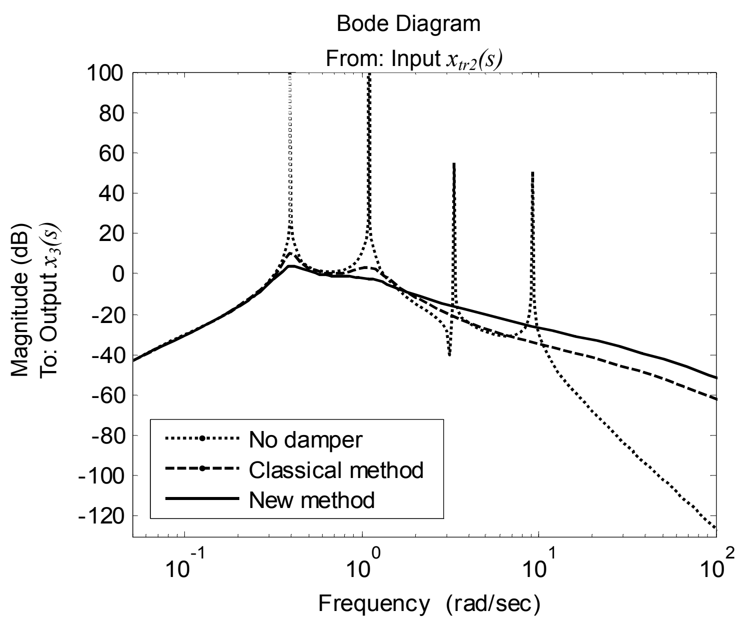


**Fig. 4.3. Gain of frequency response of $x_3(s)/x_{tr2}(s)$**

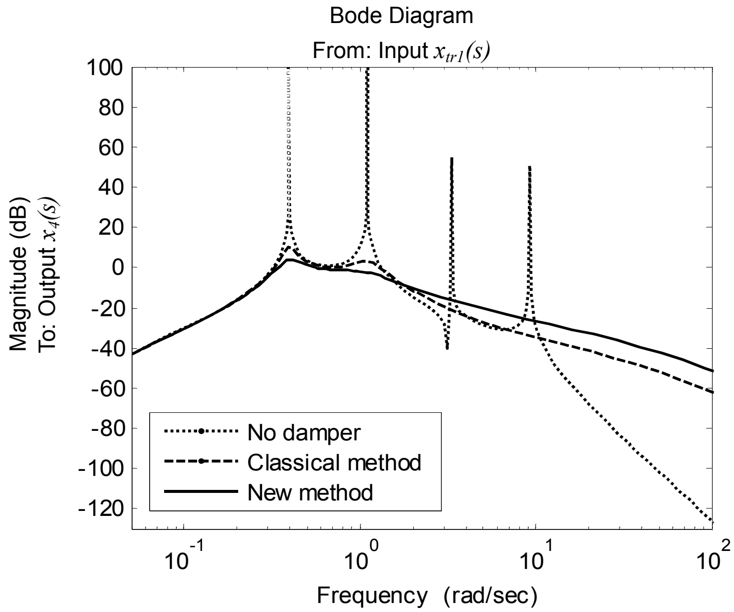


**Fig. 4.4. Gain of frequency response of $x_4(s)/x_{tr1}(s)$**

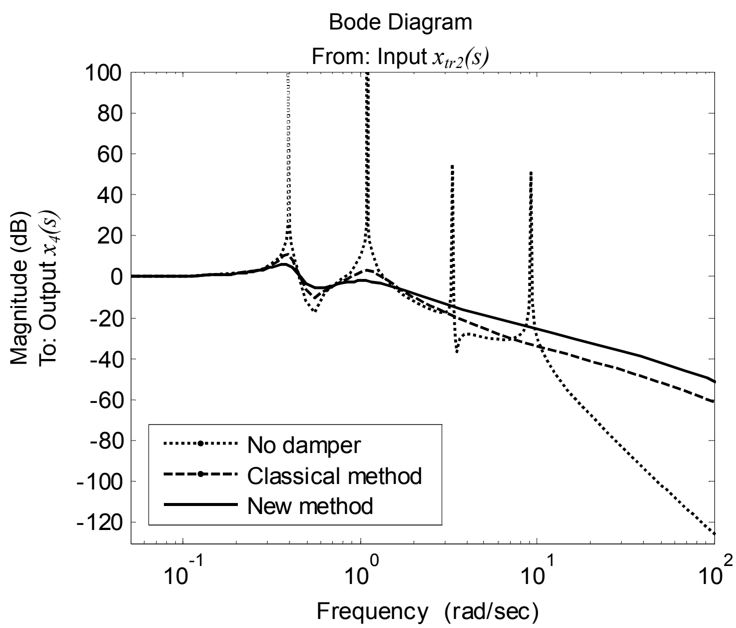


**Fig. 4.5. Gain of frequency response of $x_4(s)/x_{tr2}(s)$**

Our model is symmetric thus the outputs also have symmetric property. Therefore, we can see that Fig. 4.2 looks like Fig. 4.5 and Fig. 4.3 looks like Fig. 4.4.

The displacement at right side of train body is the combination of $x_3(s)$ excited by $x_{tr1}(s)$ and $x_3(s)$ excited by $x_{tr2}(s)$ and at the left side of train is the combination of $x_4(s)$ excited by

$x_{tr1}(s)$ and $x_4(s)$ excited by $x_{tr2}(s)$.

The goal of this design is the selection of parameters of system so that the peaks of frequency response curve at resonance frequency are lowest.

From these figures, we can obviously see the peaks of solid curves are lower than dash curves. This shows that the results of design by using control theory are better than classical method. In other words, the performance of this system which was designed by control theory is significantly better than that of the designed system by classical method.

## 4.3 Summary

By investigating a passive suspension system with six-degree-of-freedom, this chapter wants to confirm that the degree of freedom of system can be increased until the model of passive suspension system approaches to real-life situations of degree of freedom.

Moreover, it has been shown by simulations that the peaks of frequency response curve which was plotted by results of new method are lowest at resonance frequency. It shows that the performance of this system which was designed by control theory is significantly better than that of the designed system by classical method.

On the other hand, the limitation of applying control theory in design suspension systems is solving BMI problems. Although this BMI problem can be solved by using alternative algorithm, but none of the algorithms can be guaranteed to converge to a local optimum or a stationary point, it depends on the choice of initial values. Its convergence maybe very slow or even never happens. In this chapter, the initial values for alternative algorithm in solving BMI problems also are the values of parameters which are optimized by using fixed point theory. Therefore, the algorithm may converge quickly.

This chapter elucidated clearly the strong and weak points of applying control theory in design passive suspension system of multi-degree-of-freedom and compared to classical design method.

# Chapter 5
# Robust design with Two States of Two-DOF Passive Suspension System

One of the important factors to be considered in designing an engineering system is uncertain parameters of system, which emanates from natural randomness, limited data, or limited knowledge of systems. Designing a system that provides a good operation in most common uncertain parameters of the passive suspension system is required. This chapter proposes a design method of two-degree-of-freedom passive suspension system with robust performance in two states of body, full and empty load. The passive suspension system is also investigated from the view of feed back control theory. This chapter also confirms to be able to design the two-degree-of-freedom passive suspension system with robustness by using control theory in particular and ability of utilizing control theory in design robustness of multi-degree-of-freedom passive suspension system in general. This is one of strong points of applying control theory in design passive suspension system comparing to classical design method.

## 5.1 Problem Formulations

The proposed model of half railway suspension system has multiple masses: bogie mass *m'* and body mass $m_1$, $m_2$ corresponding to state 1 and state 2 as illustrated in Fig. 5.1. These mass are connected in series by springs $k_1$, $k_2$ and dampers $c_1$, $c_2$. $x_0$ is displacement of rail track excitation.

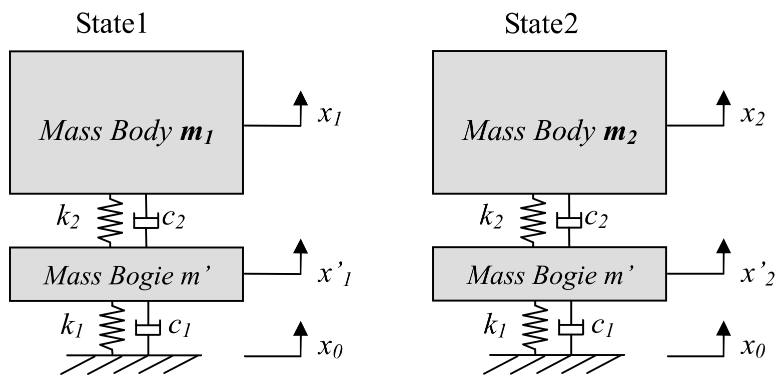


**Fig. 5.1. Two states of mass of proposed system**

In this model, the motion of train body, bogie and train wheel can be simultaneously translational up and down in paper space. $x'_i$, $x_i$ are translational motions at bogie and body respectively.

The equations of motion in the state *i* can be written as:

$$M_i\ddot{X}_i + KX_i + C\dot{X}_i = Eu_i + F_p x_0 + F_v \dot{x}_0$$

where: $M_i$, $K_i$, $C_i$ are positive definite equivalent mass, stiffness and damping matrices of state *i* respectively, $u_i=K_kY_i$ is control force, $\dot{x}$ and $x$ are vectors of velocity excitations and displacement excitations:

$$M_i = \begin{bmatrix} m & 0 \\ 0 & m_i \end{bmatrix};\ X_i = \begin{bmatrix} x'_i \\ x_i \end{bmatrix};\ K = \begin{bmatrix} k_1 & 0 \\ 0 & 0 \end{bmatrix};\ C = \begin{bmatrix} c_1 & 0 \\ 0 & 0 \end{bmatrix};\ F_p = \begin{bmatrix} k_1 \\ 0 \end{bmatrix};\ F_v = \begin{bmatrix} c_1 \\ 0 \end{bmatrix};$$

$$u_i = K_k Y_i;\ E = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix};\ K_k = \mathrm{diag}\left(k_2 \quad k_2 \quad c_2 \quad c_2\right);\ Y_i = \left[x'_i \quad x_i \quad \dot{x}'_i \quad \dot{x}_i\right]^T;$$

Designing with robustness performance is examined in the change of body weight corresponding to state 1 and state 2 of vehicle load.

The equations of motion for these two states can be rewritten as:

$$M_s \ddot{X} + K_s X + C_s \dot{X} = E_s u + F_{ps} x_0 + F_{vs} \dot{x}_0 \tag{5.1}$$

where

$$M_s = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}; X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}; K_s = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}; C_s = \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix};$$

$$E_s = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}; F_{ps} = \begin{bmatrix} F_p \\ F_p \end{bmatrix}; F_{vs} = \begin{bmatrix} F_v \\ F_v \end{bmatrix}; u = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix};\ Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix};$$

$$u = K_c Y;\ Y = \left[Y_1 \quad Y_2\right]^T;\ K_c = \mathrm{diag}(\ K_k \quad K_k\ );$$

By defining the state variable as:

$$x_s = \left[X \quad \dot{X} - M_s^{-1} F_{vs} x_0\right]^T \tag{5.2}$$

The equation of motion can be written in state-space form as:

$$\dot{x}_s = A x_s + B_1 x_0 + B_2 u \tag{5.3}$$

where $A = \begin{bmatrix} 0 & I \\ -M_s^{-1} K_s & -M_s^{-1} C_s \end{bmatrix}; B_1 = \begin{bmatrix} M_s^{-1} F_{vs} \\ M_s^{-1}\left(F_{ps} - C_s M_s^{-1} F_{vs}\right) \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ M_s^{-1} E_s \end{bmatrix}$

We write the vector of "measured" outputs, with the relative displacements and velocities at suspension connections as a linear combination of the states and inputs, that is:

$$Y = C_2 x_s + D_{21} x_0 \tag{5.4}$$

where
$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; D_{21} = \begin{bmatrix} 0 \\ 0 \\ T_1 M_s^{-1} F_{vs} \\ T_2 M_s^{-1} F_{vs} \\ 0 \\ 0 \\ T_3 M_s^{-1} F_{vs} \\ T_4 M_s^{-1} F_{vs} \end{bmatrix};$$

$$T_1 = [1 \quad 0 \quad 0 \quad 0]; T_2 = [0 \quad 1 \quad 0 \quad 0]; T_3 = [0 \quad 0 \quad 1 \quad 0];$$
$$T_4 = [0 \quad 0 \quad 0 \quad 1];$$

We can write the vertical displacement of train body in two states ($x_1$, $x_2$) as an output vector $z$, which can be expressed in the form:

$$z = C_1 x_s + D_{11} x_0 \tag{5.5}$$

where:

$$C_1 = G C_2; D_{11} = G D_{21}; G = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix};$$

The forces generated by the suspension springs and dampers are determined from $Y$ according to: $u = K_c Y$ where the "feedback gain" $K_c$ is a decentralized matrix (block-diagonal) composed of the suspension parameters to be optimized. In this problem $K_c$ is shown as:

$$K_c = \text{diag}\left(K_k \quad K_k\right) \tag{5.6}$$

Equations (5.3), (5.4) and (5.5) cast the design of the suspension system of train as a decentralized control problem, as indicated by the diagram shown in Fig. 2.4. Based on this formulation, we use decentralized control techniques $\mathcal{H}_\infty$ to directly optimize the stiffness and damping coefficients of springs and dampers to achieve performance (measured by $z$) under the disturbance of $x_0$. The goal of solving this problem is to determine the feedback law:

$$u = K_c Y \tag{5.7}$$

The feedback gain $K_c$ is a decentralized matrix composed of the parameters to be designed, and all parameters of springs and dampers which need to be designed are designable by determining $K_c$.

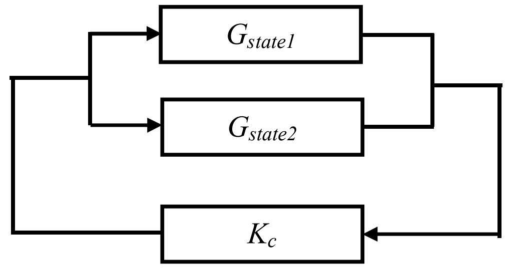


**Fig. 5.2. Concept of block diagram of proposed system**

## 5.2 Robust Design of Two-DOF Passive Suspension System of Railway Vehicle with Two States

Given parameters of scale model of railway vehicle are shown in Table 5.1. The parameters of primary stage were determined by classical method.

**Table 5.1 Given parameters of railway vehicle**

| | | | | | |
|---|---|---|---|---|---|
| $m_{t}$' | 3,000 | Kg | $k_1$ | 2,200,000 | N/m |
| $m_1$ | 14,000 | Kg | $c_1$ | 201,080 | Ns/m |
| $m_2$ | 20,000 | Kg | | | |

To solve BMI problems by alternative algorithm, we have chosen initial values for the alternative algorithm. In our problem initial values are the values which are optimized by using fixed point theory in above classical design method. Initial values for alternative algorithm are shown in Table 5.2.

**Table 5.2 Initial values for alternative algorithm**

| | | | | | |
|---|---|---|---|---|---|
| $k_2$ | 3,459,500 | N/m | $c_2$ | 55,799 | Ns/m |

Start with a stabilizing $K_s$ and repeat OP1 and OP2 until $\gamma$ can not decrease any more then by the results of obtained iterations when $\gamma$ does not decrease, we obtain the final parameters [14]. Final design parameters are shown in Table 5.3.

**Table 5.3 Final design parameters**

| | | | | | |
|---|---|---|---|---|---|
| $k_2$ | 3,555,300 | N/m | $c_2$ | 82,100 | Ns/m |

Figures 5.3, 5.4 show frequency response outputs corresponding to two states of model. The two solid-curves are the frequency responses of system in state 2 with parameters which were designed by using classical method (higher curve) and with parameters which were designed by using control technique (lower curve). Two center-curves are the frequency responses of system in nominal state with parameters which were designed by using classical method (higher curve) and with parameters which were designed by using control technique (lower curve). Two dash-curves are the frequency responses of system in state 1 with parameters which were designed by using classical method (higher curve) and with parameters which were designed by using control technique (lower curve).

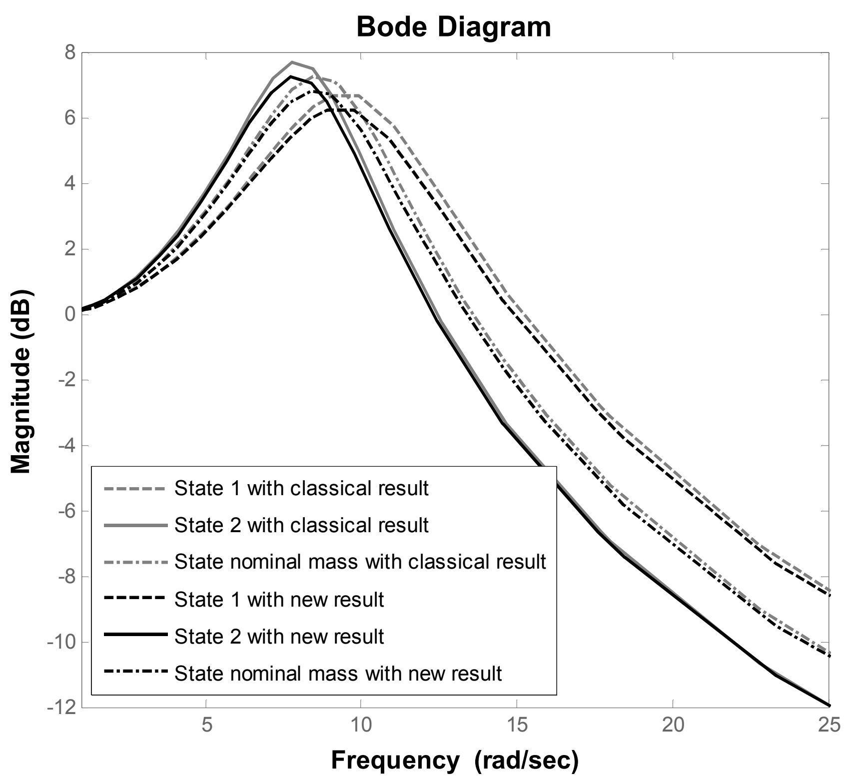


**Fig. 5.3. Frequency responses by change in mass**

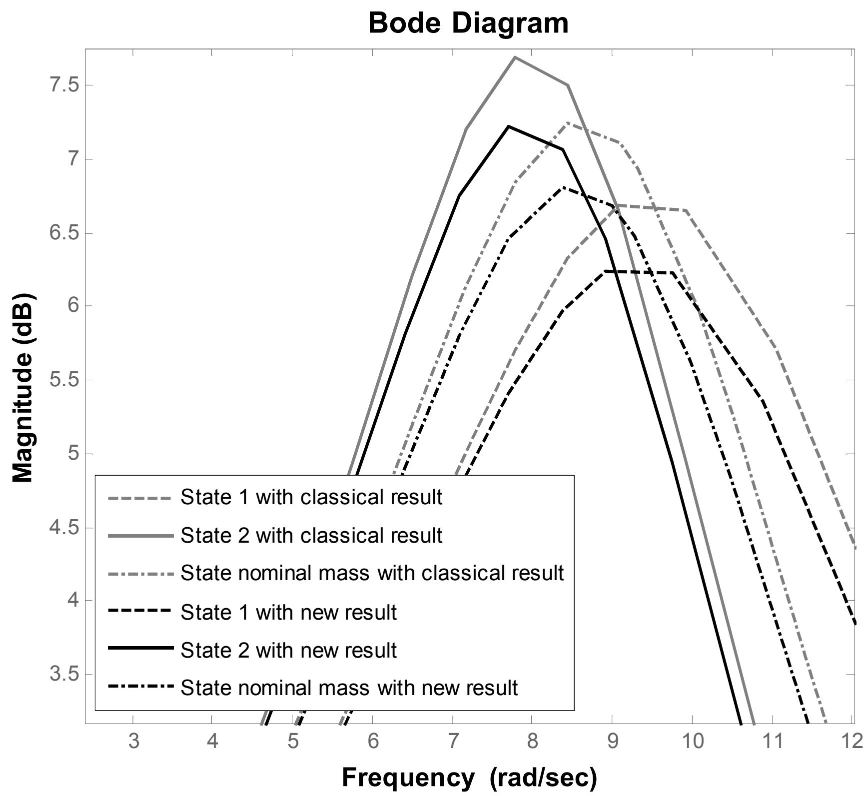


**Fig. 5.4. Close up of frequency responses by change in mass**

The goals of this design is the selection of parameters of system so that the peaks of frequency response curve at resonance frequency are lowest and also prove that by applying control theory, robust design of passive suspension system can be performed easily with multi-degree-of-freedom and in multi-states of system.

From these figures, we can obviously see the peaks of curves which have parameters designed by control technique are lower than the curves which have parameters designed by classical method in the case of nominal mass. The performance of this system which was designed by control theory is significantly better than that of the designed system by classical method in the case of nominal mass.

# 5.3 Summary

This chapter proposes a design method of two-degree-of-freedom passive suspension system with robustness performance in two states of body, full and empty load. The passive suspension system is also investigated from the view of feed back control theory. This chapter confirms to be able to design the passive suspension system with robustness performance by using control theory, not only in design the multi-degree-of-freedom passive suspension system. This is one of strong points of applying control theory in design passive suspension system comparing to other methods.

# Chapter 6
# Conclusions and Recommendations

The purpose of this chapter is to summarize this thesis and to determine weak and strong points of each method that was utilized. The chapter ends with recommendations for future work in the field of railway vehicle suspensions.

## Conclusions and Recommendations

This study focuses on design of passive suspension systems of railway vehicle. Chapter 2 and chapter 3 provide two design methods and its application in two-degree of freedom passive suspension system: a classical method which utilities fixed-points theory in optimizing parameters and a new method which utilities control theory in optimizing parameters. In new design method, passive suspension design is equivalent to design of a controller with decentralized architecture and additional constrains on the symmetry of the vehicle and the ranges of the design parameters, therefore, many difficult problems in passive mechanical systems become tractable in the framework of structural control. These chapters show that applying feedback control theory in designing a passive suspension system is one of solutions to avoid foregoing limitations of classical method. By giving a comparison between the results and applicable abilities of two methods, these chapters expressed the weak and strong points of each method.

Chapter 4 presents an applicable ability of new method in design passive suspension system of six-degree-of-freedom. By investigating a passive suspension system with six-degree-of-freedom, this chapter proves that the degree of freedom of system can be increased until the model approaches to real-life situations. This is one of strong points of applying control theory in design passive suspension system comparing to classical design method.

With the purpose of optimization in design a passive suspension system is to suppress the peaks of the magnitude of the vehicle body's frequency response at resonance frequency. We can see that the performance of system which was designed by control theory is significantly better than that designed by classical method.

At high frequency range, we can not avoid that the performance of system which is designed by new method is worse than performance of system which is designed by classical method and even worse than performance of system has no dampers, because this method just suppresses the peaks of frequency response curve at resonance frequency but does not in all frequency range.

Chapter 5 discusses about an important factor in designing an engineering system is uncertainty of some parameters, which emanates from natural randomness, limited data, or limited knowledge of systems. Thus designing a passive suspension system that provides a good operation in most common uncertain parameters of the passive suspension system is necessary to develope. This chapter proposes a design method of two-degree-of-freedom passive suspension system with robust performance in two states of body weight, full and empty load of body. The passive suspension system is also investigated from the view of feed back control theory. This chapter confirms to be able to design the two-degree-of-freedom passive suspension system with robustness by using control theory in particular and ability of utilizing control theory in design robustness of multi-degree-of-freedom passive suspension system in general. This is also one of strong points of applying control theory in design passive suspension system comparing to classical design method.

With our proposed robust design method, two equations of motion that express these states were extended as one control object and one controller designed by the static output $\mathcal{H}_\infty$ feedback control theory controlled the extended model. Thus, the designed suspension system could be controlled two simultaneous states, and suspension system had robustness performance about two states.

On the other hand, the limitation of applying control theory in design suspension systems is solving BMI problems. There are many researchers in control community have investigated the decentralized $\mathcal{H}_\infty$ optimization and various techniques have been proposed to search for the local minima of BMI problems but none of the algorithms can be guaranteed to converge to a local optimum or a stationary point, it depends on the choice of initial values, its convergence was slow or even never happens. In this paper, the BMI problems were solved by using alternative algorithm, the initial values for alternative algorithm are the values of parameters which are optimized by using fixed point theory. It has been clear that the results of design by using classical method are quite close to optimal parameters. Therefore, the algorithm may converge quickly.

For further research it is necessary to develop a new algorithm that can search for the local minima of BMI problems and it need to be guaranteed to converge to a local optimum.

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