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Vibration Control of Structure by using Tuned Mass Damper
(Development of system which suppress displacement of auxiliary mass)

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Abstract

In vibration control of a structure by using an active tuned mass damper (ATMD), stroke of the auxiliary mass is so limited that it is difficult to control the vibration in the case of large disturbance input. In this paper, two methods are proposed for the problem. One of the methods is a switching control system by two types of controllers. One of the controllers is normal controller under small relative displacement of the auxiliary mass, and the other is not effective only for first mode of vibration under large relative displacement of the auxiliary mass. New variable gain control system is constructed by switching these two controllers. The other method is the brake system. In active vibration control, it is necessary to use actuator for active control. By using the actuator, the proposed system puts on the brake to suppress displacement increase of the auxiliary mass under large disturbance input. Finally, the systems are designed and the effectiveness of the systems is confirmed by the simulation.

Key Words: *Vibration control; variable gain controller; brake system*

1. Introduction

In recent years, in order to give high habitability and safety to flexible structures like a tall building, the response control of structures by using active tuned mass damper (ATMD) has been proposed and its control effect has been investigated experimentally and analytically, and also it is already applied to practical structure.[1~4] However, stroke of the auxiliary mass of ATMD is usually so limited because of the limitations of the actuator and the floor space. Therefore, although the system shows good performance in the case of small disturbance input such as wind and small earthquake, the auxiliary mass bumps into the stopper in the case of large disturbance input. Then performance of vibration control for the system is likely to get worse or the system is likely to get unstable. In present system with ATMD applied to the practical structure, in order to prevent such phenomena, it is considered that the auxiliary mass moves passively or the system uses another controller that is not effective for all vibration modes of structure.[1]

In this paper, two methods are proposed to prevent the problem. First method is to use a new switching controller. We notice that the stroke of the auxiliary mass becomes especially large at first mode of vibration of the structure. Then, two types of controllers are prepared. One of the controllers is effective for all vibration modes of structure under small displacement of the auxiliary mass, and the other is not very effective only for first vibration mode of structure under large displacement of the auxiliary mass. The variable gain control system is considered by switching these two controllers. Both controllers are based on H^{∞} control theory which has ability of designing frequency-shaped control system.[2),6)] Until now, the controller which is not very effective for whole frequency region has been used in case of large disturbance input.[1)] Therefore, the performance of our proposed control system is compared with conventional one. Moreover, we consider relative displacement of the auxiliary mass as parameters, and prepare thresholds given theoretically for each parameter as a border of switching controllers. In this way, we design the variable gain control system which can be applied to relatively large disturbance input, while the performance of vibration control (especially for high vibration mode of structure) is almost equal to the simple effective controller for all modes.

Second method is a brake system. In this paper, we propose new brake system by using an actuator for vibration control. Generally, at least one actuator is used to control in active vibration control. For example, AC motor, DC motor, Linear motor, and so on.[1~4)] By using these actuators, we propose a new system which puts on the brake to suppress displacement increase of the auxiliary mass under large disturbance input. When the damping coefficient of the tuned mass damper increases from the optimal value, the damping force, which is obtained by increase of the coefficient, acts like the brake force. Then, assuming that the damping coefficient is increased, the variable damping coefficient is represented by an exponential function. Furthermore, the vurtual damping force is realized by these actuators. We also notice timing of the brake. Usually, in order to avoid excess of displacement limit of the auxiliary mass, the brake is put on just before getting to the maximum point of displacement of the mass. Considering action of the brake, it is important to use high speed of the auxiliary mass, because the brake force is made by the damping force. Therefore, it is more effective to put on the brake around the equivalent point of displacement of the mass and the maximum point of velocity of the mass. Furthermore, the brake force comes from actuator, the force acts to stop the auxiliary mass and not to increase an amplitude of vibration of the structure. Therefore, the system is stable.

Then the effectiveness of these systems is confirmed by the simulation with four-degree-of-freedom model of the structure.

2. Structural Model

In this study, the four-degree-of-freedom (4DOF) structural model with tuned mass damper (TMD) mounted on top floor is considered as shown in Fig.1. In this system, x_i ($i = 1, 2, 3, 4$) is the relative displacement of the i-th mass to the ground, $\ddot{z}_g$ is the input acceleration of earthquake. m_a, c_a, k_a are mass, damping coefficient and stiffness of the TMD, while c_a and k_a are determined so that they almost correspond to those obtained from Den Hartog's PQ theorem.[5)] The control force u is generated on the spring part of the TMD. Assuming that linear motor is used as an actuator for the auxiliary mass, and considering of dynamics of linear motor, control force u is described as follows.

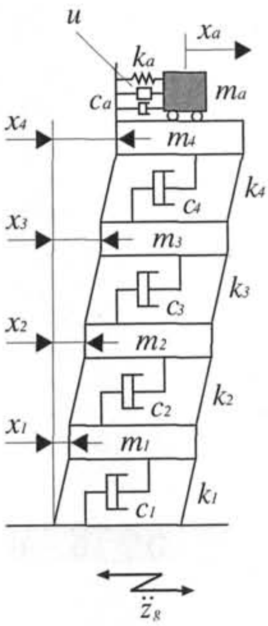

Fig. 1. 4DOF structural model with TMD.

$$u(t) = -\frac{K_e K_{rf}}{R}\dot{x}_a(t) + \frac{K_{rf}}{R}V(t) \tag{1}$$

where K_{rf}, K_e are thrust constant and voltage constant of linear motor, R, $V(t)$ are resistance and control voltage of equivalent circuit of linear motor. Then, applying Newton's second law to the 4DOF structural model with TMD and summing up the equations, the following equations of motion of the structure and TMD system are obtained.

$$\boldsymbol{M}_f \ddot{\boldsymbol{x}}_f + \boldsymbol{C}_f \dot{\boldsymbol{x}}_f + \boldsymbol{K}_f \boldsymbol{x}_f = \boldsymbol{E}_f \ddot{z}_g + \boldsymbol{F}_f V(t) \tag{2}$$

Using the state vector $\boldsymbol{x} = \left[\boldsymbol{\eta}\, x_a\, \dot{\boldsymbol{\eta}}\, \dot{x}_a\right]^T$, the following state equation is given.

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}V + \boldsymbol{d}_g \ddot{z}_g \tag{3}$$

where $\boldsymbol{\eta} = [\eta_1 \dots \eta_4] = \Phi_s^{-1}\boldsymbol{x}_s$, and Φ_s is the modal matrix normalized as $\Phi_s^T M \Phi_s = \boldsymbol{I}$, and

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0}_{5\times5} & \boldsymbol{I}_{5\times5} \\ -\Phi^T \boldsymbol{K}\Phi & -\Phi^T \boldsymbol{C}\Phi \end{bmatrix}, \quad \boldsymbol{B} = \left[\boldsymbol{0}_{1\times5} \quad (\boldsymbol{M}_f^{-1}\boldsymbol{F}_f)^T\right]^T$$

$$\boldsymbol{d}_g = \left[\boldsymbol{0}_{1\times5} \quad (\boldsymbol{M}_f^{-1}\boldsymbol{E}_f)^T\right]^T, \quad \Phi = \begin{bmatrix} \Phi_s & \boldsymbol{0}_{4\times1} \\ \boldsymbol{0}_{1\times4} & 1 \end{bmatrix}$$

We consider $\left(\ddot{x}_4 + \ddot{z}_g\right)$ as the output of the system, that is the absolute acceleration of the top floor mass.

$$y = \boldsymbol{C}x + \boldsymbol{D}V + \boldsymbol{H}_g \ddot{z}_g \tag{4}$$

where

$$C = [\,0\;0\;0\;1\;0\,]\Phi\left[-\Phi^T K\Phi \;-\Phi^T C\Phi\right]$$

$$D = [\,0\;0\;0\;1\;0\,]\Phi\Phi^T F$$

$$H_g = [\,0\;0\;0\;1\;0\,]\Phi\Phi^T E + 1$$

Thus, the controllers for the model described by Eq.(3) and Eq.(4) are designed in the next section.

3. Design of H^∞ Control System

In this section, the following two types of controllers based on H^∞ control theory[3),6)] is designed for variable gain control system.

1. The controller which is effective for all vibration mode of structure
2. The controller which is not very effective for only first vibration mode

In this paper, above two controllers are termed controller1 and controller2, respectively.

In addition, in order to compare our proposed system with conventional one,[7)] we also design the controller, which is not very effective for all vibration mode of structure, and this controller is used instead of contoller2.

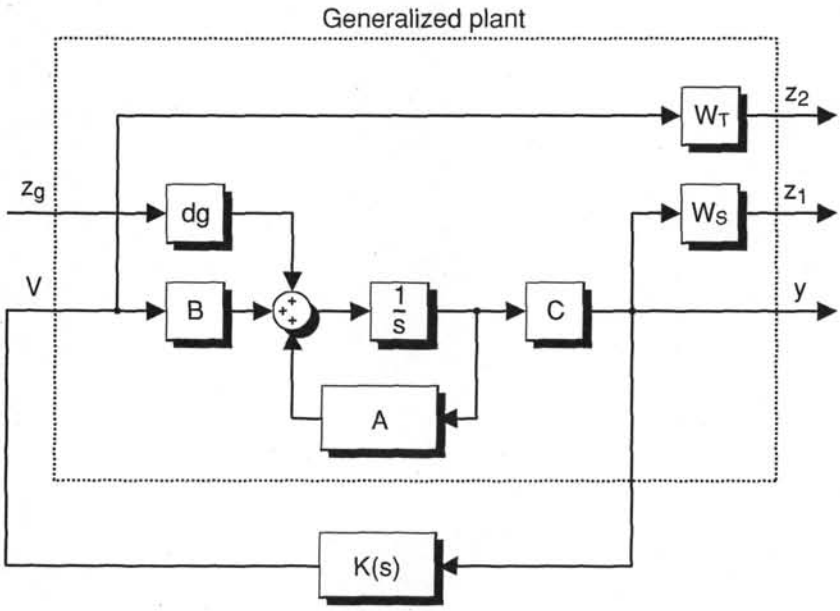

Fig. 2. Generalized plant.

Figure 2 shows the generalized plant of the model described in previous section. In this figure, y is the observation output (absolute acceleration of 4th mass of the structure), V is the control voltage, $z = [z_1\, z_2]^T$ is the controlled output, and $w = \ddot{z}_g$ is the disturbance input. W_S and W_T are the frequency weight functions for performance of vibration control and for guarantee of stability robustness. Now, describing state-space representation of W_S and W_T as follows,

$$W_S : \begin{cases} \dot{x}_s = A_s x_s + B_s y \\ z_s = C_s x_s + D_s y \end{cases}$$

$$W_T : \begin{cases} \dot{x}_t = A_t x_t + B_t V \\ z_t = C_t x_t + D_t V \end{cases}$$

the following generalized plant including weight functions are obtained.

$$P(s) = \left[\begin{array}{ccc|cc|c} A_s & 0 & B_s C & 0 & 0 & 0 \\ 0 & A_t & 0 & 0 & 0 & B_t \\ 0 & 0 & A & d_g & 0 & B \\ \hline C_t & 0 & D_s C & 0 & 0 & 0 \\ 0 & C_t & 0 & 0 & 0 & D_t \\ \hline 0 & 0 & C & 0 & 1 & 0 \end{array}\right] \tag{5}$$

We consider of designing H^∞ output feedback controllers for generalized plant in Eq.(5), then H^∞ norm constraint is described by mixed sensitivity problem as follows.

$$\left\| \begin{bmatrix} W_S S \\ W_T T \end{bmatrix} \right\|_\infty < 1 \tag{6}$$

where

$$S = [I - P_0 K]^{-1} P_w$$

$$T = K[I - P_0 K]^{-1} P_w$$

$$P_0 = C(sI - A)^{-1} B + D$$

$$P_w = C(sI - A)^{-1} d_g + H_g$$

To design controllers, the compensator K(s) that fills the condition in Eq.(6) must be obtained.

We carried out the computations for designing two types of H^∞ controllers with software MATLAB. Figure 3 shows the transfer function of the system with each controller by simulation. Here, it is obviously that the controller1 is effective for all modes of structure, and controller2 is not very effective for the first mode of structure. Controller3, which is used instead of controller2 as conventional variable gain control system, is not very effective for all modes of structure.

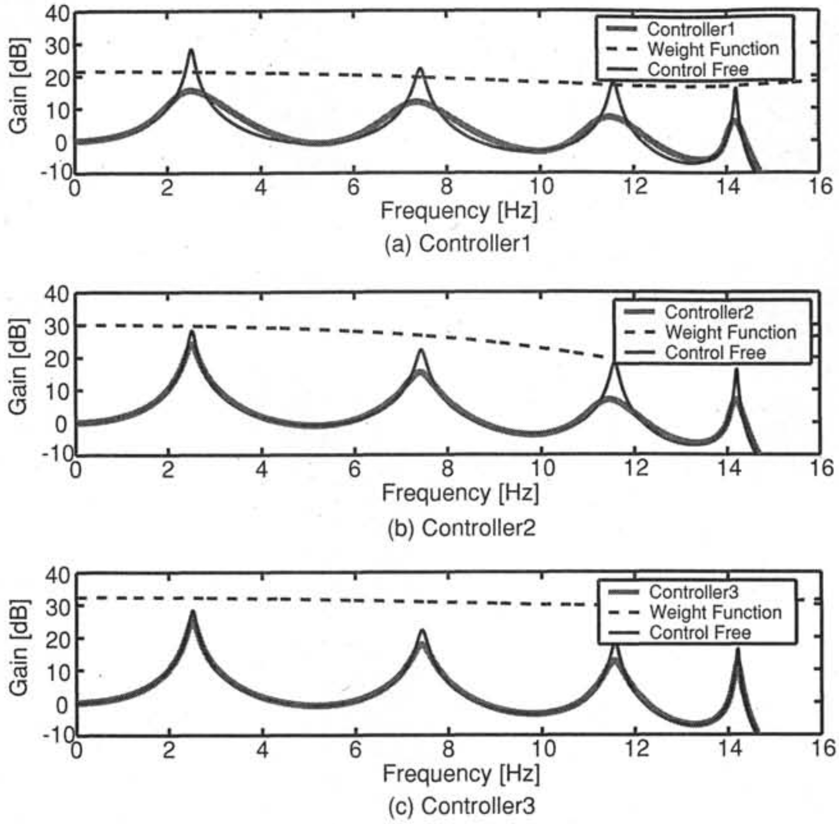

Fig. 3. Design of H^{∞} Controllers (Transfer Function by (a)Controller1 (b)Controller2 (c)Controller3).

4. Switching Controllers

As mentioned in introduction, our purpose is to design a variable gain control system by switching controller1 and controller2 designed in previous section. Then, as shown in Fig.4, we consider the relative displacement of the auxiliary mass as the parameter of horizontal axis, prepare thresholds x_{at} given as a border of switching controllers, and use controller1 and controller2 under the condition $|x_a| \leq x_{at}$ (field1) and $|x_a| > x_{at}$ (field2), respectively. However, as the controller is suddenly switched from controller1 to controller2, control input is discontinued. Therefore, the intermediate field between the field for controller1 and the field for controller2 is prepared. In the intermediate field, the control voltage is given as follows,

$$V = \frac{b - |x_a|}{b} V_1 + \frac{|x_a|}{b} V_2 \tag{7}$$

where b is the width of intermediate field, V_1 , V_2 are control voltages obtained by only controller1 or controller2.

In this way of switching controllers, we compare performance of proposed control system and conventional control system by simulation using random noise input under the condition that the same $x_{at} = 4[mm]$ and $b = 1[mm]$ are used respectively, and the same disturbance input to the each system. Figure 5 shows transfer functions from disturbance input to the acceleration of 4th mass of structure. Figure 6 also shows the displacement response of the auxiliary mass by the simulation. From these results, it is shown that the

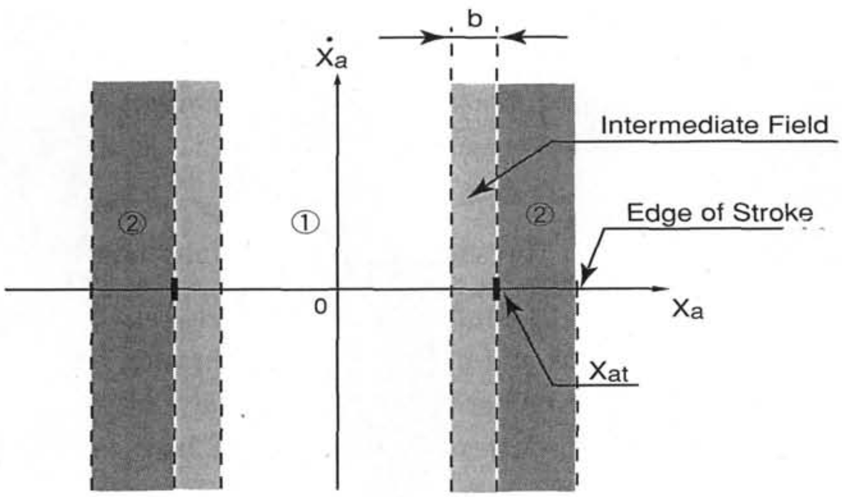

Fig. 4. Method of switching controllers evaluating only x_a.

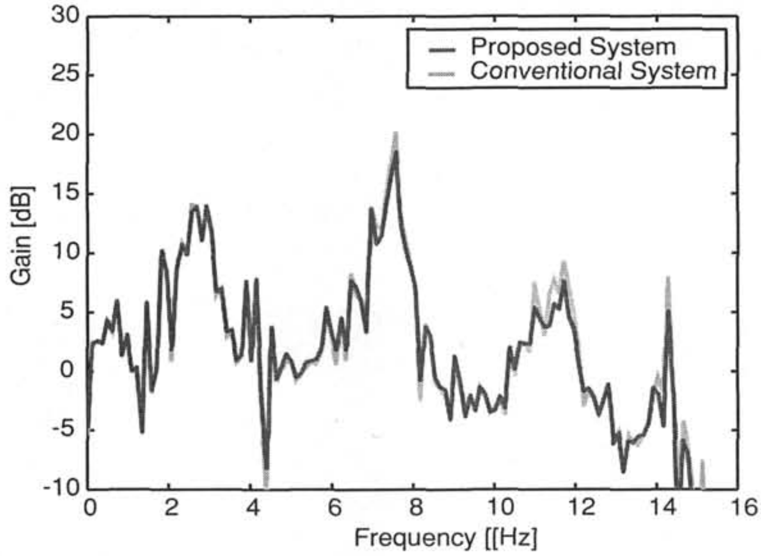

Fig. 5. Comparison between transfer function of proposed system and conventional system.

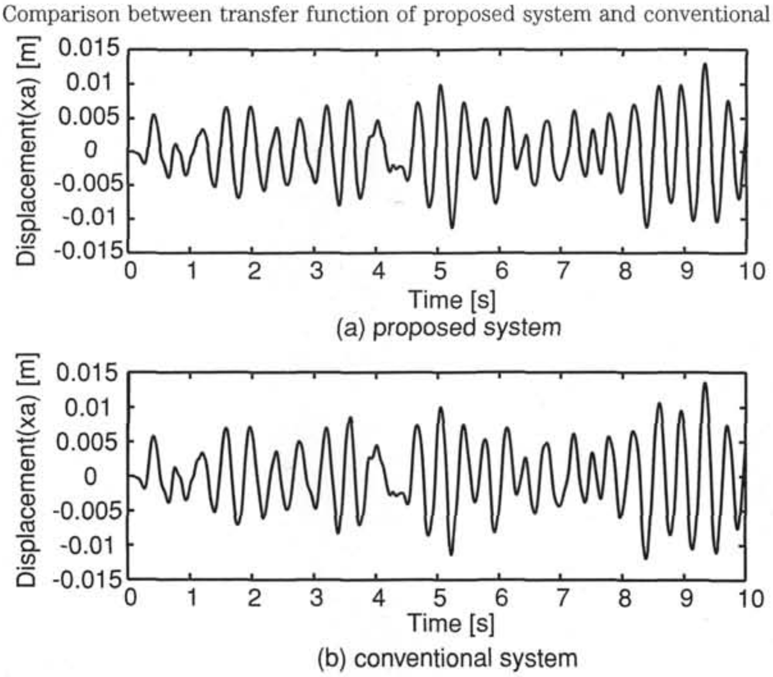

Fig. 6. Comparison between displacement(x_a) of (a) proposed system and (b) conventional system.

third and fourth resonant peaks of proposed system are more suppressed than that of conventional system, though there is not much difference in the maximum displacement of the auxiliary mass. It is found that proposed variable gain control method exhibits better performance of vibration control than conventional one.

5. Method of Deciding Threshold and Width of Intermediate Field

In the proposed variable gain control system of previous section, the relative displacement of the auxiliary mass and performance of vibration control depend on the threshold and the width of intermediate field. Accordingly, referring to Fig.4, as the threshold becomes smaller, field1 becomes narrow and field2 becomes wide. Therefore, the relative displacement of the auxiliary mass becomes smaller, however the performance gets worse. On the contrary, as the threshold becomes larger, the performance gets better, however the relative displacement of the auxiliary mass becomes larger. Here, it is obviously that there is a trade off between maximum relative displacement of 4th mass of the structure and the maximum relative displacement of the auxiliary mass. As a result of the numerical calculation, the relations between each of them and threshold described dotted lines in Fig.7 are obtained. In this figure, scales of x_4 and x_a are so different that each value of the vertical axis of the graph is defined dimensionless as follows,

$$
\begin{aligned}
x_{4dl} &= \left\{ \frac{\max(x_4)}{\min(x_{4max})} - 1 \right\} \times r \\
x_{adl} &= \frac{\max(x_a)}{\min(x_{a\,max})} - 1 \\
x_{4max} &= \max(x_4)^{(i)} \quad (i = 2 \ldots 12) \\
x_{a\,max} &= \max(x_a)^{(i)} \quad (i = 2 \ldots 12) \\
r &= \frac{m_a}{m_4}
\end{aligned}
$$

where x_{4dl} and x_{adl} are dimensionless $\max(x_4)$ and $\max(x_a)$, $\max(x_4)^{(i)}$ and $\max(x_a)^{(i)}$ are maximum value of x_4 and x_a at the threshold $x_{at} = i$. And then, we decide the threshold at the intersection of these two lines as a compromise one. Next, using the compromise threshold, we mention about the width of intermediate field. In the similar way, considering the same trade off, the relations between x_{4dl}, x_{adl} and width of intermediate field relations can be described dotted lines in Fig.8. Finally, we decide the width of intermediate field at the intersection of these two lines as a compromise one.

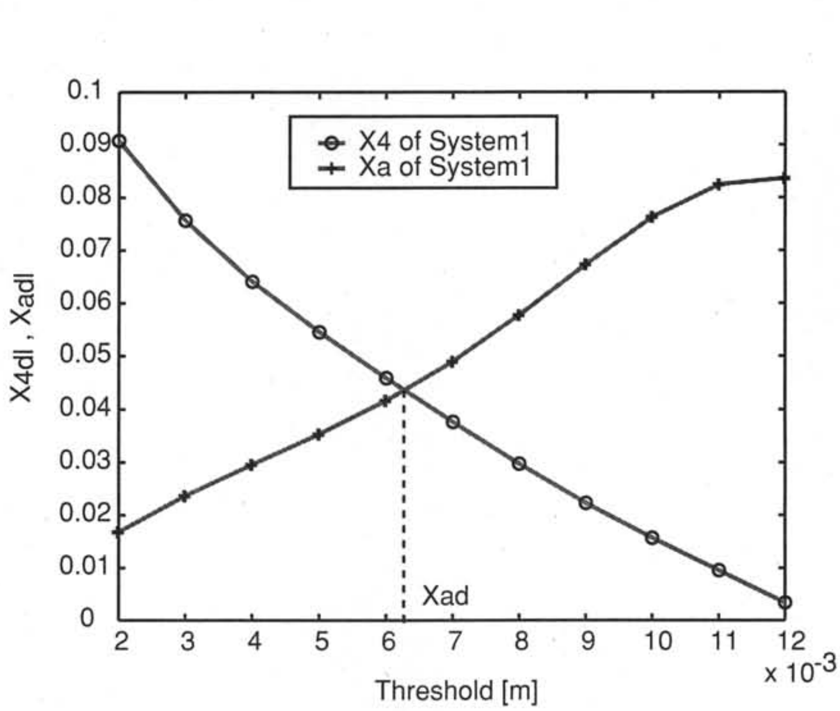

Fig. 7. Compromise threshold.

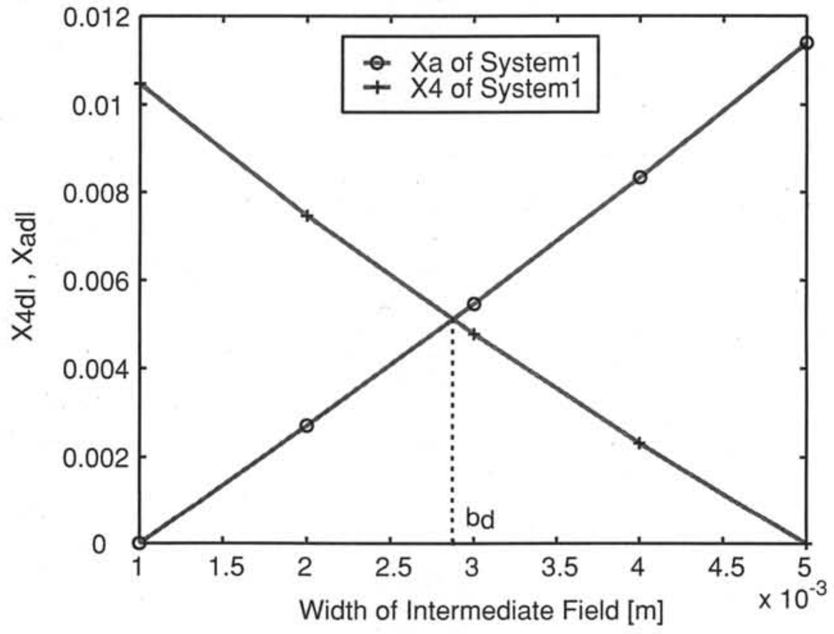

Fig. 8. Compromise width of intermediate field.

6. Brake System

In this section, we explain new brake system by using actuator for vibration control. In this study, a Linear motor is used to control in active vibration control. By using the Linear motor, the proposed system puts on the brake to suppress increase of relative displacement of the auxiliary mass under large disturbance input. Previously, in order to stop the auxiliary mass around the limitation of displacement, a method was considered that the stiffness of TMD is increased near the maximum displacement point of the mass.[8] However the control performance becomes worse by slightly change of the stiffness as shown in Fig.9. Then, in this study, new method is proposed that the damping coefficient of TMD is increased to consume motion energy of the auxiliary mass. When the damping coefficient of the tuned mass damper increases from the optimal value, the damping force, which is obtained by increase of the coefficient, acts like the brake force. In this case, the control performance doesn't get spoiled by slightly change of the damping coefficient as shown in Fig.9.

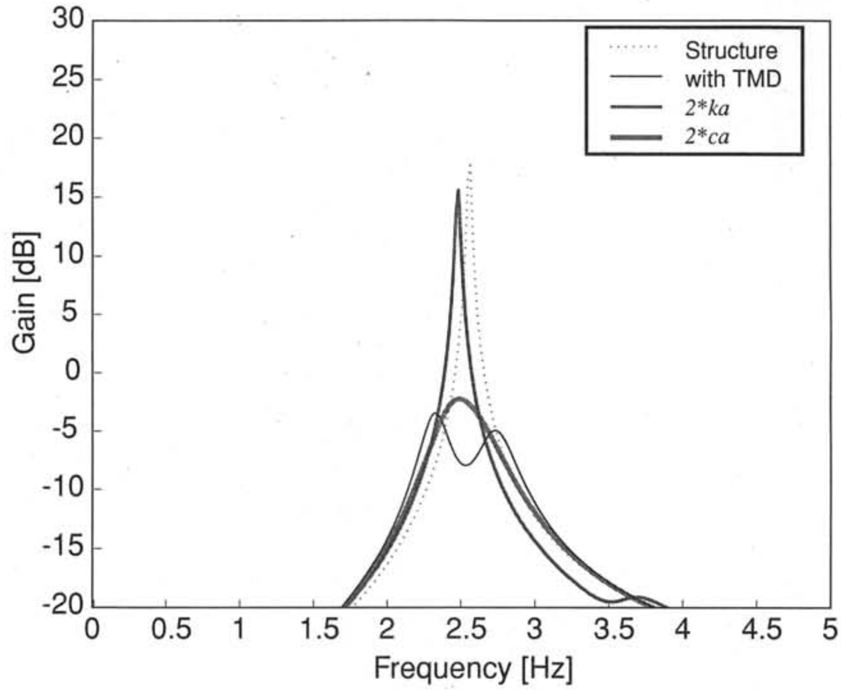

Fig. 9. Comparison of transfer functions about variable factors.

Usually, in order to avoid excess of displacement limit of the auxiliary mass, the brake is put on around the maximum point of displacement of the mass. Considering action of the brake system by using virtual damping coefficient, it is important to use high speed of the auxiliary mass. Therefore, it is more effective to put on the brake around the equivalent point of displacement of the mass and the maximum point of velocity of the mass. Then, in the new system, the virtual damping coefficient of TMD is increased near the maximum velocity point of the mass as shown in Fig.10, where the maximum velocity point is considered as the maximum amplitude of velocity at the right moment of the maximum amplitude of displacement.

The product of the increasing part of damping coefficient and the velocity of the auxiliary mass is the braking force. In this study, the virtual damping coefficient of TMD is represented by using the following exponential function which is related to relative velocity of the auxiliary mass as shown in Fig.10.

$$p_2(\dot{x}_a) = \cosh(24.3\,\dot{x}_a) \tag{8}$$

In order to compare, the other virtual damping coefficient is represented by the following exponential function which is related to relative displacement of the mass.

$$p_1(x_a) = \cosh(355\,x_a) \tag{9}$$

The maximum damping coefficients are fifty times as much as the optimal damping coefficient as shown in Fig.10 and Fig.11. The brake force is represented by the following equation.

$$u_{brake} = (p_2(\dot{x}_a) - 1)\, c_a\, \dot{x}_a \tag{10}$$

By equation (1), the brake force, which is obtained from the change of damping coefficient, becomes the following output voltage of the Linear motor.

$$V = \frac{(p_2(\dot{x}_a) - 1)\, c_a R + K_e K_{rf}}{K_{rf}}\, \dot{x}_a \tag{11}$$

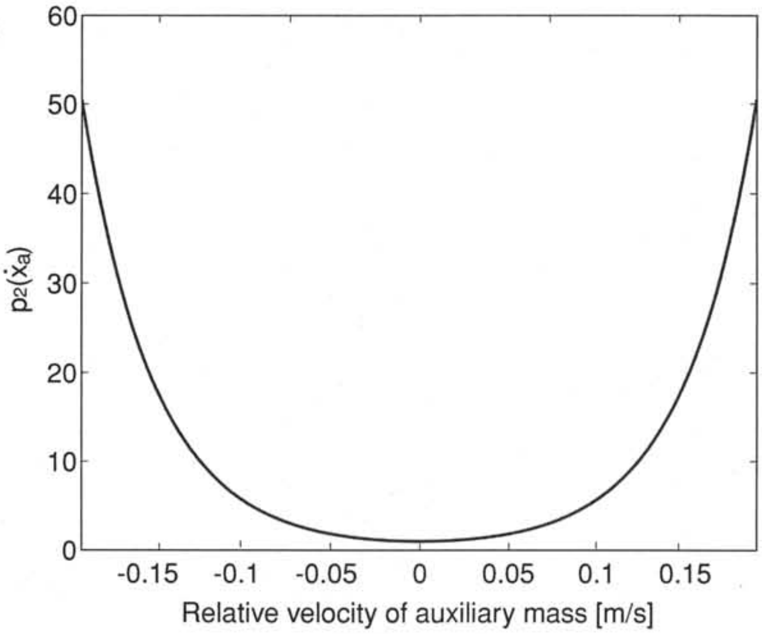

Fig. 10. Exponential functions $p_2(\dot{x}_a)$ for stroke suppression.

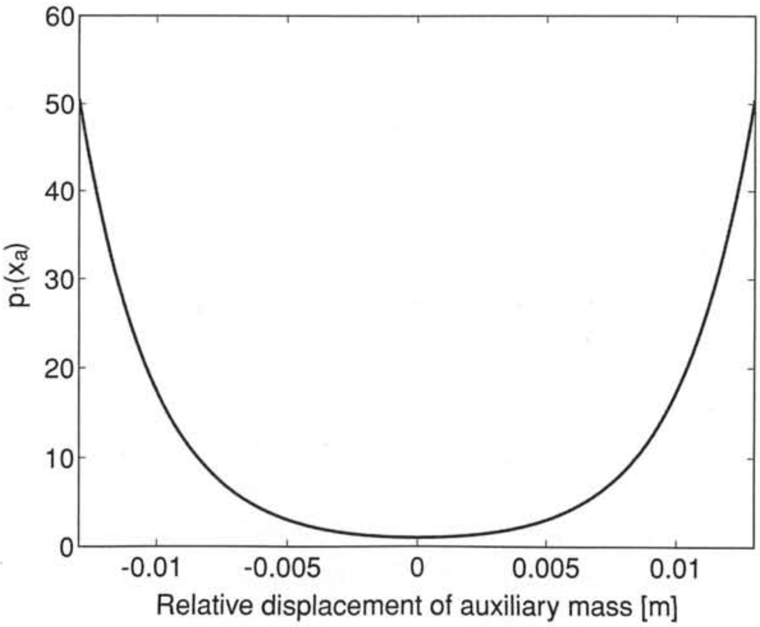

Fig. 11. Exponential functions $p_1(\dot{x}_a)$ for stroke suppression.

By using eq.(11), the brake force is obtained from the Linear motor for active control.

The effectiveness of this brake system is confirmed by the simulation with the 4DOF structural model. The following figures shows the response of the structure with this brake system. In this simulation, impulse disturbance is inputted to the foundation of the structure, and the active vibration control isn't performed. In order to compare the proposed systems, the results using each exponential functions, which are considered relative displacement and relative velocity of the auxiliary mass, are shown in the following figures.

Figure 12 shows velocity response of the 4th floor as the control performance. From this figure, we can see that the control performance gets worse, because of the brake force. Figures 13 and 14 shows the responses of relative velocity and relative displacement of the auxiliary mass. From these figures, it is clear that the amplitude of relative velocity and relative displacement are suppressed by the brake system. Figure 15 shows the time history of the exponential functions $p_1(x_a)$, $p_2(\dot{x}_a)$. The product of the function value and the optimal damping coefficient and relative velocity of the auxiliary mass becomes the braking

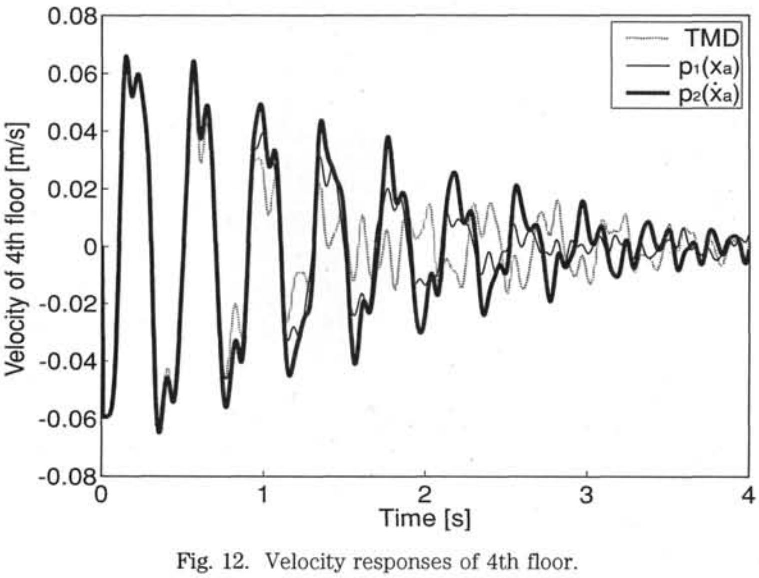

Fig. 12. Velocity responses of 4th floor.

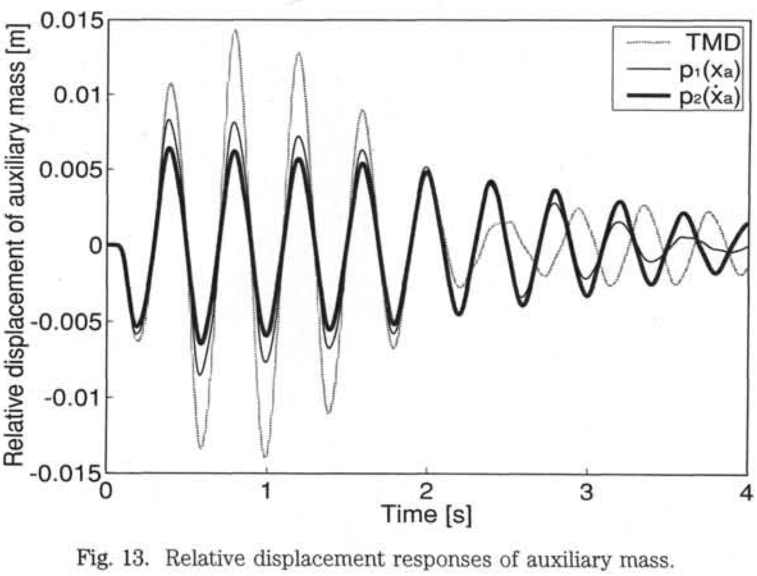

Fig. 13. Relative displacement responses of auxiliary mass.

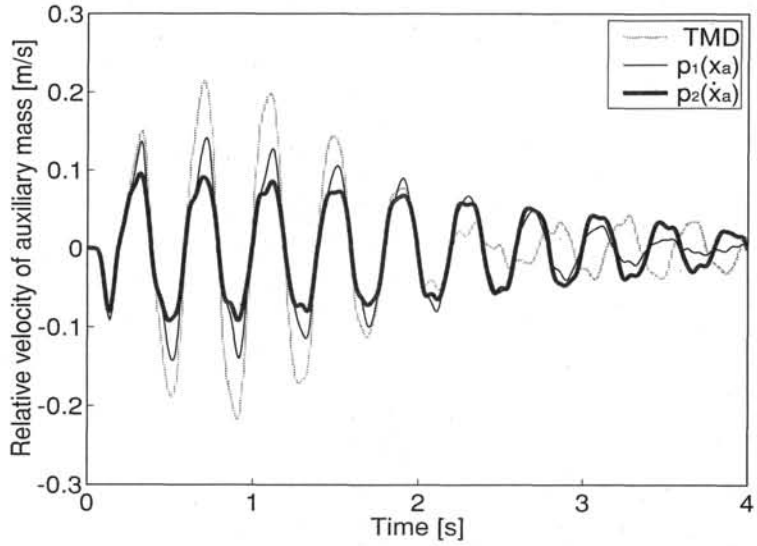

Fig. 14. Relative velocity responses of auxiliary mass.

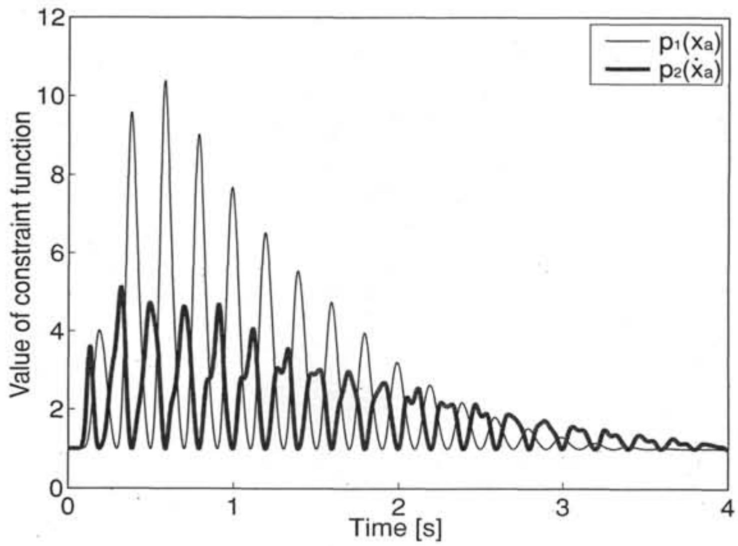

Fig. 15. Time history of $p_1(x_a), p_2(\dot{x}_a)$.

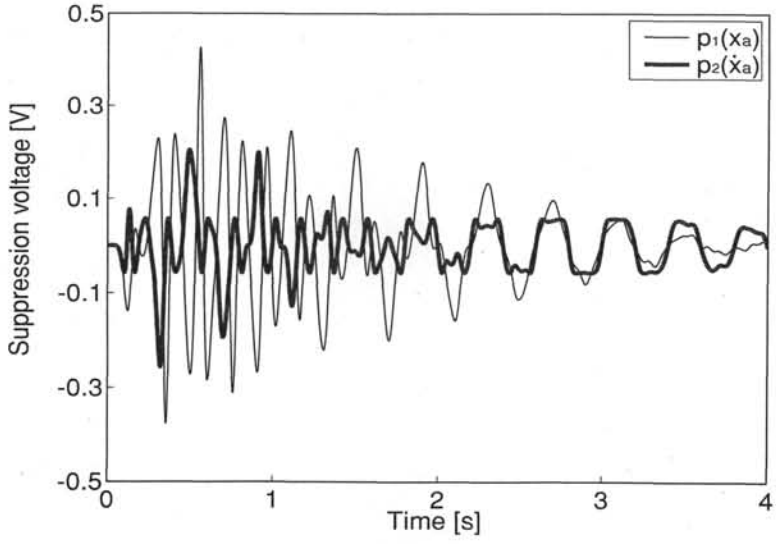

Fig. 16. Suppression voltages.

force $u_{brake} = (p_2(\dot{x}_a) - 1)c_a\dot{x}_a$. The brake force is converted into the voltage of the Linear motor as shown in Fig.16. From this figure, it is obviously that the voltage which is considered the exponential function related to relative velocity of the mass is smaller than that of relative displacement of the mass. Namely, it is more effective to use the exponential function related to relative velocity of the mass.

Furthermore, the brake force comes from actuator, the force act to stop the auxiliary mass and not to increase an amplitude of vibration of the structure. Therefor, the system is stable.

7. Conclusions

In this study, we proposed two methods to suppress displacement of the auxiliary mass. One of the methods is the switching controller by two types of controllers. First, we prepare two H^∞ controllers, namely one of the controllers is effective for all vibration modes of structure and the other is not very effective only for first vibration mode of structure. Then, the method of switching these controllers depending on displacement of the auxiliary mass is proposed. The other method is the brake system. By using the actuator for control, the proposed system puts on the brake to suppress increase of relative displacement of the auxiliary mass under large disturbance input. The effectiveness of two proposed methods is confirmed by the simulation. From the results of simulations, it is found that the proposed systems exhibit good performance, and can be applied to the large disturbance input.

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